DEVELOPMENT OF PATH FOLLOWING AND COOPERATIVE MOTION CONTROL ALGORITHMS FOR AUTONOMOUS UNDERWATER VEHICLES

Thesis Submitted to

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of

Doctor of Philosophy

by

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Under the Supervision of

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June 2015

"Dedicated To My Loving Parents"

CERTIFICATE

This is to certify that the thesis entitled DEVELOPMENT OF PATH FOLLOWING AND COOPERATIVE MOTION CONTROL ALGORITHMS FOR AUTONOMOUS UNDERWATER VEHICLES, submitted by MR. BASANT KUMAR SAHU to National Institute of Technology Rourkela, is a record of bonafide research work pursued under my supervision and I consider it worthy of consideration for the award of the degree of Doctor of Philosophy of the Institute. The results embodied in the thesis have not been submitted for the award of any other degree or diploma elsewhere.

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Acronyms

APD	Augmented Proportional Derivative
APF	Attractive Potential Function
AUV	Autonomous Underwater Vehicle
DALF	Distance-Angle leader-follower
DDLF	Distance-Distance leader-follower
DOF	Degrees of Freedom
DOM	Degree of Membership
EKF	Extended Kalman Filter
FLM	Fuzzy Logic Method
FLV	Fuzzy Linguistic Variable
FMF	Fuzzy Membership Functions
FRB	Fuzzy Rule Base
FRP	Formation Reference Point
FRPF	Fuzzy Repulsive Potential Function
GCPD	Gravity compensation Proportional Derivative
HRT	Hybrid Reference Type
IET	Imaginary Equilateral Triangle
IOS	Input-to-State stability
KF	Kalman Filter
KWP	Key Way Points

LBL	Long Base Line
LFS	Leader-to-Formation Stability
LOS	Line of Sight
LRT	Leader Reference Type
MIQP	Mixed Integer Quadratic Programming
MRPF	Mathematical Repulsive Potential Function
NRT	Neighbour Reference Type
ODIN	Omni Directional Intelligent Navigator
PD	Proportional Derivative
PFAPD	Potential Function based Augmented Proportional Derivative
PFPD	Potential Function based Proportional Derivative
PRM	Probabilistic Random Manner
PRT	Predecessor Reference Type
RPF	Repulsive Potential Function
SOM	Self-Organizing Map
SRT	Single Reference Type
TMF	Triangular Membership Functions
UUV	Unmanned Undersea Vehicles

List of Symbols

m	Mass of AUV
I_Z	Rotational mass
$X_{\dot{u}}$	Added mass
$Y_{\dot{v}}$	Added mass
$N_{\dot{r}}$	Added mass
X_u	Surge linear drag
$X_{u\|u\|}$	Surge quadratic drag
Y_v	Sway linear drag
$Y_{v\|v\|}$	Sway quadratic drag
N_r	Yaw linear drag
$N_{r\|r\|}$	Yaw quadratic drag
$\{B\}$	Body fixed frame of reference
$\{I\}$	Inertial frame of reference
η_i	Position and orientation vector of i^{th} AUV
ν_i	Velocity vector
u_i, v_i, w_i	Linear velocity vectors
p_i, q_i, r_i	Angular velocity vectors
X_i, Y_i, Z_i	Force vector

K_i, M_i, N_i Moments

M_i Inertia matrix including added mass

$C_i(\nu_i)$ Matrix of Coriolis and centripetal terms

$D_i(\nu_i)$ Hydrodynamic damping and lift matrix

$g_i(\eta_i)$ Vector of gravitational forces and moments

τ_i Vector of forces

$J_i(\eta_i)$ Velocity transformation matrix

r_{ij} Distance between i^{th} and j^{th} AUVs

d Desired distance between i^{th} and j^{th} AUVs

n_l Number of leader AUVs

n_f Number of follower AUVs

η_i^c Flocking centre of i^{th} AUV

η_c Average of position coordinates

η^c Vector of flocking centres

ε Error between ζ^c and average position coordinate of all AUVs

V Lyapunov function

$U(\eta)$ Potential function

$U_{i,att}^c\left(\eta_i, \eta_i^c\right)$ Attractive potential function between i^{th} AUV and its flocking centres

$F_{i,att}^c$ Force of attraction between the position of i^{th} AUV and position of corresponding flocking centre

$F_{i,att}^{d}$ Force of attraction between the position of i^{th} AUV and position of desired path

F Total potential function of the whole system

η_i^l Position and orientation vector of i^{th} leader AUV

η_i^f Position and orientation vector of i^{th} follower AUV

η_i^{lc} Flocking centre of the i^{th} leader AUV

η_i^{fc} Flocking centre of the i^{th} follower AUV

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Abstract

Research on autonomous underwater vehicle (AUV) is motivating and challenging owing to their specific applications such as defence, mine counter measure, pipeline inspections, risky missions e.g. oceanographic observations, bathymetric surveys, ocean floor analysis, military uses, and recovery of lost man-made objects. Motion control of AUVs is concerned with navigation, path following and co-operative motion control problems.

A number of control complexities are encountered in AUV motion control such as nonlinearities in mass matrix, hydrodynamic terms and ocean currents. These pose challenges to develop efficient control algorithms such that the accurate path following task and effective group co-ordination can be achieved in face of parametric uncertainties and disturbances and communication constraints in acoustic medium. This thesis first proposes development of a number of path following control laws and new co-operative motion control algorithms for achieving successful motion control objectives. These algorithms are potential function based proportional derivative path following control laws, adaptive trajectory based formation control, formation control of multiple AUVs steering towards a safety region, mathematical potential function based flocking control and fuzzy potential function based flocking control.

Development of a path following control algorithm aims at generating appropriate control law, such that an AUV tracks a predefined desired path. In this thesis first path following control laws are developed for an underactuated (the number of inputs are lesser than the degrees of freedom) AUV. A potential function based proportional derivative (PFPD) control law is derived to govern the motion of the AUV in an obstacle-rich environment (environment populated by obstacles). For obstacle avoidance, a mathematical potential function is exploited, which provides a repulsive force between the AUV and the solid obstacles intersecting the desired path. Simulations were carried out considering a special type of AUV i.e. Omni Directional Intelligent Navigator (ODIN) to study the efficacy of the developed PFPD controller. For achieving more accuracy in the path following performance, a new controller (potential function based augmented proportional derivative, PFAPD) has been designed by the mass matrix augmentation with PFPD control law. Simulations were made and the results obtained with PFAPD controller are compared with that of PFPD controller.

From comparison of the above performances, it is observed that the proposed PFAPD controller is able to drive the AUV to track accurately the desired path avoiding the obstacles even in an obstacle-rich environment and PFAPD controller outperform the PFPD controller.

To overcome the effect of uncertainties due to hydrodynamic coefficients and to drive the AUV along the desired path accurately, an adaptive path following controller is developed using the concept of regressor matrix which is used in designing a direct adaptive controller. The stability of the said controller law has been verified using the Lyapunov stability criterion. This controller considers the hydrodynamic parametric uncertainties in the AUV dynamics.

Tasks like pipe line survey and seafloor exploration, etc. are not possible by the applications of a single AUV. These problems can be solved by the deploying a group of AUVs as a whole which is considered as cooperative control of AUVs. Cooperative motion control of AUVs plays a major role in solving the difficult and risky tasks. There are two types of cooperative control paradigms. These are formation and flocking control of AUVs. In this thesis, cooperative motion control using formation control approach is developed first subsequently flocking control approach for multiple AUVs is developed.

The adaptive path following control law described above is extended to develop distributed formation control algorithms to drive a group of AUVs moving as a whole along their desired paths. The motion is distributed in the sense that, each AUV plans its motion based upon the task provided to track along the desired trajectory. Simulation studies are carried out considering three AUVs in a group. Obtained results demonstrate the effect of proposed formation control algorithm for a group of AUVs tracking a desired trajectory.

During the motion of AUVs, there is a chance of entering into a hazardous area in the oceanic environment. To avoid this situation, a gravity compensation proportional derivative (GCPD) formation control law based on potential functions is developed to steer a group of AUVs form an unsafe region to a safety region. The developed GCPD controller is able to provide appropriate stress on a group of AUVs to move towards a safety region avoiding collision among them during tracking. The safety region is considered as a cylindrical region, which is surrounded by unsafe region. Inter distance dependent potential functions are used to avoid collision among the AUVs. For simulation purpose a popular AUV called ODIN is considered. A group of three AUVs is considered for studying the efficacy of the developed control formation law. From the obtained results, it is found that, this control algorithm provides an effective co-operative motion for a group of AUVs to steer towards a desired safety cylindrical region.

Another method of group motion of AUVs is called flocking where a number of AUVs are deployed to perform a single task. In case of formation control, a particular shape and inter AUV distances are to be maintained. But in case of flocking, this is not required, only the AUVs need to stay within a range of prescribed distance. Another important point of difference between the formation and flocking control is that, mathematical complexities in formation are more than the flocking control problems.

A flocking control algorithm is developed for a group of AUVs. A leader-follower flocking control strategy is developed to flock a group of AUVs along a desired path. Leader-AUVs are provided with global knowledge of the desired trajectory and other AUVs (followers) have no knowledge of the desired path. A flocking centre is estimated to keep all AUVs connected in a group. This flocking centre is a virtual point whose positions at any instant of time are predicted by using a consensus algorithm and by taking the average of the position of all the AUVs in the group. The controllers for the leader and follower AUVs are developed using artificial potential functions which are the functions of the distances between the AUVs. Simulations are carried out to study the efficacy of the mathematical potential based flocking controller. From the obtained results it is concluded that a small but finite error exists in the performances. To minimize this error and to compensate for the parametric uncertainties associated with the hydrodynamic damping terms, a fuzzy potential function based flocking control algorithm is then developed to flock a group of participating AUVs along the desired path to achieve accurate path following performances. Fuzzy artificial potential functions are calculated by using Mamdani's fuzzy logic technique. A group of four AUVs is considered for studying the efficacy of the proposed fuzzy flocking control algorithm. Simulations are carried out both in obstacle free and obstacle rich environment. Results obtained by fuzzy potential based flocking fuzzy flocking control algorithm envisage that effective co-operative motion control of multiple AUVs is achieved along a desired path with obstacle avoidance. It is also found that this flocking controller developed based on fuzzy artificial potential function exhibits superior control performance than the controller with mathematical potential functions in terms of improved path following and obstacle avoidance.

The thesis contributes towards developing a new path following controller through mass matrix augmentation with potential function based proportional derivative law and accurate path following performance is achieved in an obstacle-free as well as obstacle-rich environments. Subsequently the thesis explores to design an adaptive path following controller using the concept of regressor matrix and Lyapunov's theory. After having developed of path following controller, a number of cooperative control algorithms in formation and flocking paradigms such as adaptive formation control, GCPD formation control, mathematical potential function based flocking control, fuzzy potential function based flocking control laws have been developed. In path following task, it is observed that PFAPD performs better than that of PFPD and in flocking problem fuzzy potential function based flocking control laws outperforms the mathematical potential function based flocking control laws. Further flocking control needs less computational time than the formation control.

Key Words: Autonomous underwater vehicle, Formation control, Multi-robot system, Multi-agent system, Multi AUV system, Under actuated system, Potential functions, Obstacle avoidance, Path following, Trajectory tracking, Path generation, Mesh stability, Ring stability, Leader to formation stability, Coordination strategy, Graph theory, Centralized control, Decentralized control, Distributed control.

Contents

Chapter 1

Introduction

1.1 Background

Autonomous underwater vehicle (AUV) is an unmanned mobile robot which is deployed in oceanic environment. These are used in military applications as unmanned undersea vehicles (UUVs). AUVs constitute a large part of undersea system known as unmanned underwater vehicles. A non-autonomous remotely operated underwater vehicle (ROV) is controlled and powered from the surface by an operator/pilot via an umbilical.

AUVs find specific applications such as security patrols, search and rescue in hazardous environments [1], [2], [3]. In military missions, a group of autonomous vehicles is required to keep a specified formation for area coverage and reconnaissance. A group of AUVs can be put in a cooperation mission through the formation or flocking which are deployed in different applications. Exploring and developing deep sea areas needed different kinds of sensors and devices. Remotely operated vehicles (ROVs) and AUVs are directly fulfilling these requirements [4]. AUV plays an important role in case of high resolution seabed mapping and commercial survey. In aquaculture, AUVs are used to feed the fishes [7].

1.2 Applications of AUVs

Recently AUVs has been used for a limited number of tasks dictated by the technology available. With the development of more advanced processing capabilities and high yield power supplies, AUVs are now being used for more and more tasks with roles and missions constantly evolving. AUVs play a crucial role in the exploration and exploitation of resources located in deep oceanic environments. In case of exploration and exploitation of resources located in deep ocean environment, AUVs play important roles. The AUVs are also used in risky and hazardous operations such as bathymetric surveys, oceanographic observations, recovery of lost man-made objects and ocean floor analysis, etc. They are employed in risky missions such as oceanographic observations, bathymetric surveys, ocean floor analysis, military applications, recovery of lost man-made objects, etc. [2], [3]. Meanwhile, due to mini AUV's characters of small bulk, good invisibility, low energy consumption, low cost, flexibility, batch production and good manoeuvrability, it has been one of the directions in AUV development.

1.2.1 Commercial

The oil and gas industries employ AUVs to make detailed maps of the seafloor before they start building subsea infrastructure; pipelines and subsea completions can be installed in the most cost effective manner with minimum disruption to the environment. AUVs allow survey companies to conduct precise surveys or areas where traditional bathymetric surveys would be less effective or too costly. Also, post-lay pipe surveys are now possible through applications of AUVs.

1.2.2 Military

A typical military mission for an AUV is to map an area to determine if there are any mines, or to monitor a protected area (such as a harbour) for new unidentified objects. AUVs are also employed in anti-submarine warfare, to aid in the detection of manned submarines.

1.2.3 Collection of Oceanic Information

AUVs are used to study lakes, the ocean, and the ocean floor. A variety of sensors can be attached to AUVs to measure the concentration of various elements or compounds, the absorption or reflection of light, and the presence of microscopic life. The researchers conjecture that platoons of cooperating mobile robots or autonomous vehicles provide significant benefits over single- unit approaches for a variety of tasks. Further, cooperating AUVs do not need to be sophisticated or expensive to outperform many advanced independent units for tasks such as material transport and scouting, etc. [8].

In the applications described before, a group of AUVs is required to follow a predefined trajectory while maintaining a desired spatial pattern. Moving in formation has many advantages over conventional motion plan, for example, it can reduce the system cost, increases the robustness and efficiency of the system while providing redundancy, reconfiguration ability and flexibility for the system [1].

Other applications of formation control are security patrols, search and rescue in hazardous environments. In military missions, a group of AUVs are required to keep in a specified formation for area coverage and reconnaissance; in small satellite clustering formation helps to reduce the fuel consumption for propulsion and expand their sensing capabilities [9], In automated highway system (AHS), the throughput of the transportation network can be greatly increased if the vehicles can form platoons at a desired velocity while keeping a specified distance between vehicles [8].

1.3 Path Following Control of an AUV

The motion control of AUV is categorized as point stabilization, trajectory tracking and path following (Figure 1.1).

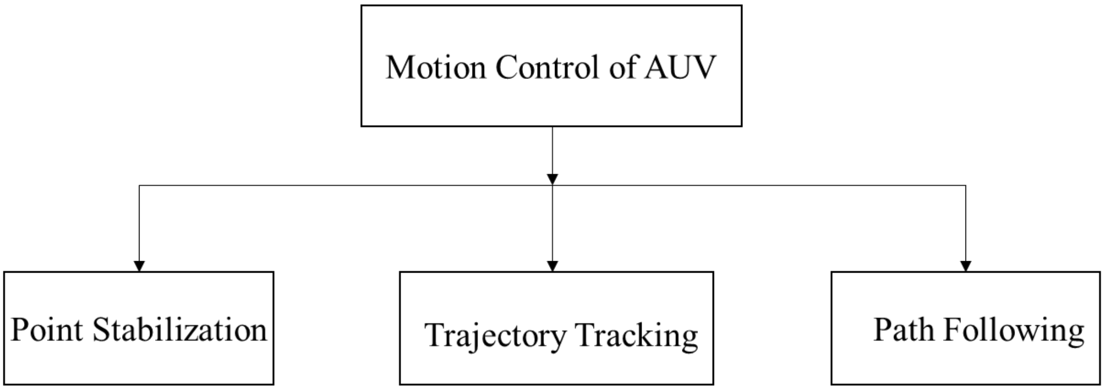


Figure 1.1: Methods of motion control of AUV

Point stabilization: Point stabilization problem deals with the motion of AUV to reach a final target point. It require a particular desired orientation.

Trajectory Tracking: Trajectory tracking deals with the problem of time constraint navigation of an AUV. AUV has to track a time parametric reference path.

Path Following: The path following control problems of AUVs deals with tracking the motion along the desired predefined path. In this, an AUV starts from an initial point and reach at the final point with a definite orientation. Sometimes controlling orientation of the AUV may be avoided according to the necessary degrees of freedom and the directions are controlled by the planner orientations. Here the accuracy is the major concern during the motion along the desired trajectory. In the speed control problems, the forward speed are forced to the desired speed, but in case of the position control problems the positions are directed to merge with the desired positions of the desired path.This thesis concentrates on the path following control of AUV.

1.4 Review on Path Following Control of AUV

This section presents a review on path following control approaches for AUVs reported in literature. In [260], the path following problem considered explicitly with the AUV dynamics. The controller is developed based on the Lyapunov's theory and the back-stepping techniques. However, this work has considered a Serret-Frenet frame analysis instead of a three dimensional one, AUV follows the motion of a virtual target. The path following controller is developed using the AUV kinematics only [161]. The output feedback concept is used to develop controller. Here an error space is developed and then linearized, then the static output feedback controller is designed considering linearized AUV model. A complicated 3D path following controller is developed in [261]. Here a sliding mode technique combined with predictive control strategy is used to develop the controller for path following of an underactuated AUV where a 3D virtual guidance orientation (VGO) approach is used to deal with the AUV dynamics. The AUV has to track a virtual point along the desired path with avoiding saturation problem of driving devices (stern plane and rudder) of the AUV. In [262], an active disturbance rejection control (ADRC) controller is developed based on support vector regression (SVR) to derive an underactuated AUV. The SVRADRC is designed to follow a path in the horizontal plane considering two dimensional planes only. However, higher dimension analysis is an important consideration in accommodating oceanic disturbance. In [263], a spatial path following controller is developed for an AUV in the presence of ocean currents. A Lyapunov's stability criterion based neural network controller is developed to drive an AUV along the desired path. The neural network controller is used to estimate the uncertain parameters due to presence of oceanic currents. Here speed and diving controllers are designed but not position controller. An AUV path following controller explicitly related to the dynamics of the AUV is developed in [264]. A Lyapunov theory based backstepping controller is developed to drive that AUV. Here Serret-Frenet frame is used explicitly. A dynamic model based robust nonlinear path following controller for an underactuated AUV is developed in [265]. The controller is developed using the Lyapunov stability theorem and backstepping techniques. Although robustness to the uncertainties associated with the AUV is considered, but it is also considered the analysis in a two dimensional plane. The path following as well as obstacle avoidance problem has been solved in 2D and 3D in [266]. Here the controllers are designed in two horizontal plane (x-y, x-z) and are combined. A Lagrange multiplier based path following controller is developed for driving the underwater robots in [203]. Here a framework for motion control algorithm is developed to drive the vehicle along the desired path.

1.5 Cooperative Control of Multiple AUVs

In a cooperative motion control paradigm, a number of AUVs are deployed for achieving successful group motion control mission (Figure 1.2). Formation and flocking control of multiple AUVs are considered as cooperative control. Cooperative control of AUVs is an important research topic. In the past few decades, a good number of publications have been reported on this topic. But confusion lies in the selection of appropriate formation techniques while applying to a group of AUVs. This is because every technique has its advantages and disadvantages. Hence, proper appraisal in these techniques is essential. This section presents a comprehensive review on cooperative control of multiple AUVs based on literature reported till now. This review depicts the different cooperative techniques such as control and cooperative strategies of multiple AUVs. A detail description and classification of various cooperative techniques are presented and advantages and disadvantages of all the techniques are identified. Different sub-problems of cooperative techniques such as collision and obstacle avoidance, cooperative shape generation, path generation are explained. Stability analysis of the feasible cooperative control is presented.

1.5.1 Formation Control of Multiple AUVs

Through formation control algorithm, the cooperative motion multi-AUV systems can be achieved. The formation control deals with as the problem of controlling the relative positions and orientations of AUVs moving in a group while allowing the group to move as a whole. The step-by-step problems of formation control are accomplished through the following distinct steps; (i) assignment of feasible formation (ii) moving in formation (iii) maintenance of formation shape (iv) switching between formations. Formation is specified in two different ways, i.e. the rigid formation [5] and flexible formation (desired configuration may vary) [6].

Figure 1.2: Cooperative motion of autonomous vehicles

[https://raweb.inria.fr/rapportsactivite/RA2010/necs/uid58.html], [http://dst.jpl.nasa.gov/control/], [http://webuser.unicas.it/arrichiello/Ricerca.htm]

1.5.2 Flocking control of multiple AUVs

Flocking is the flying behaviours of a group (flock) of birds. This is applicable to control a group of multiple AUVs to perform a desired task. Flocking control of multiple AUVs is similar to that of formation control with only difference is that there are no constraints on distance among AUVs (no distance among AUVs is zero to avoid collision). In case of formation control, the distances among AUVs are always fixed.

1.6 Review on Cooperative Control of AUVs

Inspiration of cooperative control comes from natural and biological agents such as swarming behaviour of living beings, flocks of birds, schools of fishes, herds of wild beasts and colonies of bacteria etc. It is known that, a cooperative biological behaviour has certain advantages such as; increasing the chances of finding food, avoiding predators and saving energy. Observing the flocking of birds, it is found that when they fly as a flock, they avoid collision among them with a common average heading. These wonderful features of nature emulate the mathematical understanding of collective behaviours of agents moving in a group. The fundamental aspects of flocking behaviour observed from the program *'boid'* by Reynolds are multi agent cohesion, separation, and alignment [10]. In boid, each bird uses a local control strategy to achieve a desirable overall group behaviour. If the local strategy of each bird is followed, it is found that the flying strategy consists of three modules, namely; alignment (steer towards the average heading of neighbours), separation (steer to avoid crowding) and cohesion (steer towards the average position of neighbours). A cooperative control problem considers the following typical aspects; assignment of feasible formation, maintenance of formation shape and switching between formations. Formation of a group of AUVs may be a rigid formation [5] where the formation structure remains fixed throughout the moving period or it may be a flexible formation [6] where there is chance of change of formation structure.

In many applications, a group of AUVs is intended to follow a predefined trajectory while maintaining a desired spatial pattern. The purpose of employing a number of AUVs for a group motion in formation has many advantages over a single vehicle. It increases the robustness and efficiency of the system while providing redundancy, enhance the reconfiguration ability and structure flexibility [1].

Cooperative control is used in a number of applications such as rescue and search in hazardous environments, security patrols [161]. The multi-AUV system is used for survey of the oceanographic environment at deep sea, efforts an efficient data acquisition network service in deep sea, risk-full operations under ice for exploration. A group of AUVs travel in deep sea in different levels in desired geometric fashion to cover more area in less time. In military missions, a group of autonomous air vehicles is required to keep in a specified formation for area coverage and reconnaissance, in small satellite clustering formation helps to reduce the fuel consumption for propulsion and expand their sensing capabilities. Multi-robot systems are used in toxicity residual cleaning, manipulation and transport of large objects. In automated highway system (AHS), the throughput of the transportation network can be greatly increased if vehicles move in platoons at a desired velocity while keeping a specified distance between vehicles [8].

1.7 Different Cooperative Control Techniques of AUVs

Due to increased applications of formation control in various fields, it is essential to know the control issues when multiple AUVs are in a cooperative motion. To achieve a successful formation control of multiple AUVs, a number of important steps should be followed. These steps are, choosing of AUVs to stay in formation group, destination and trajectory fixation, co-ordination strategies and control strategies. These steps are then arranged under a classification tree which is called *taxonomy* of formation structure. Formation control techniques consist of four stages, namely; selection of AUVs with targets, control abstraction, cooperative strategies and control strategies (Figure 1.3).

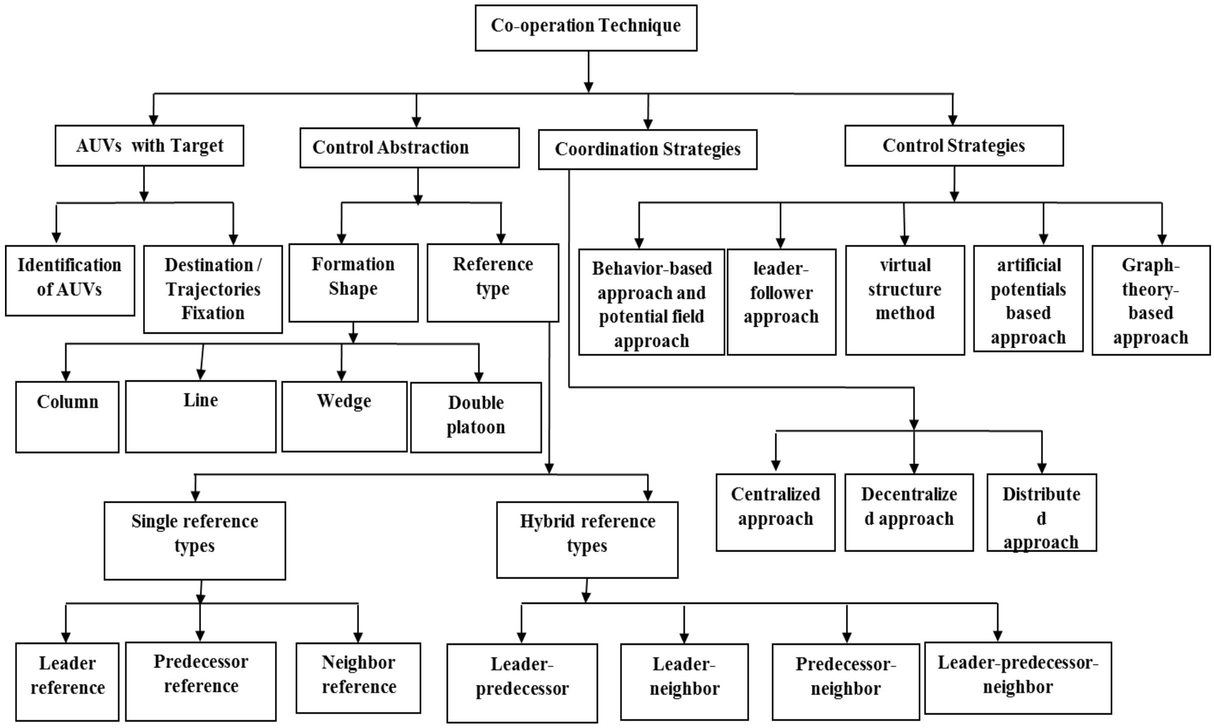


Figure 1.3: Different Cooperative and Control strategies AUVs

1.7.1 Selection of AUVs and Fixation of Targets

Before selecting the AUVs to take part in the formation task, it is necessary to choose appropriate AUVs for this. AUVs can be classified on the basis of the relative comparison between the availability of number of actuators $s(\tau)$ and the degrees of freedom (DOF) d of the AUV. Here τ is the input vector $(\tau \in R^{6\times1})$, i.e number of actuators, $s(\cdot) = size\,of\,(\cdot)$. These inputs are in terms of forces and moments.

Consider the motion of an AUV in six DOF [34].

The kinematic equation of the AUV is given by

$$\dot{\eta} = J(\eta)v \tag{1.1}$$

The dynamic equation of motion can be expressed as

$$M(v)\dot{v} + C(v)v + D(v)v + g(\eta) = \tau \tag{1.2}$$

where $\eta = [x, y, z, \phi, \theta, \psi]^T$ is the position vector of the system. x, y and z denote linear positions. ϕ, θ, ψ are angular positions. $v = [u, v, w, p, q, r]^T$ is the velocity vector in the body-fixed frame. *u*, *v* and *w* denote linear velocities. *p*, *q*, *r* are angular velocities. $\mathrm{M}(\nu)$ is the inertia matrix including added mass, $\mathrm{C}(\nu)$ is the matrix of Coriolis and centripetal terms including the added mass. $\mathrm{D}(\nu)$ denotes the hydrodynamic damping and lift matrix and $g(\eta)$ is the vector of gravitational forces and moments. $J(\eta)$ is the velocity transformation matrix between AUV and earth fixed frames. $\tau = [X,Y,Z,K,M,P]^T$ is the vector of forces and moments acting on the AUV in the body-fixed frame *X*, *Y*, *Z* are forces, *K*, *M*, *P* denote moments.

An AUV may be under-actuated (less number of actuators/inputs than that of DOF i.e $s(\tau) < d$) [175], [177], over-actuated (more number of actuators/inputs than that DOF i.e. $s(\tau) > d$) or fully actuated (equal number of actuators/inputs to that DOF i.e. $s(\tau) = d$). Selection of AUV depends on the nature of formation task to be performed.

The target of manoeuvring may be static or dynamic. For reaching at the target point, the formation-structure travels through a particular path. The formation-structure travelling problems may be path planning, path following, path generating, time varying or time invariant which are explained in section V as sub-problems of the formation control. The searching of the target, trajectory or path is known as *search task* and the tracking a found task is the *track task* [130].

1.7.2 Control Abstraction

The control abstraction of formation technique consists of two layers [32]. These are *formation* shape and *reference type*. A formation needs to maintain a specific formation shape which is explained in the first layer of *control abstraction*. In the reference type, the positions of the AUVs are computed.

1.7.2.1 Formation Shape

The desired formation shape depends upon the formation graph. A formation graph can be obtained from formation-structure considering the position of AUVs as vertices and connecting lines are edges.

An undirected formation graph G consists of a pair of sets (V,E) where V is a finite nonempty set of elements called vertices. $E(E \in V \times V)$ is the set of unordered pair of distinct AUVs called edge which denotes the communication relation between AUVs [129]. The vertex set V and the edge set E can be expressed as $V(G)$ and $E(G)$ respectively. If $i, j \in V$ and $(i, j) \in E$, then *i* and *j* are said to be adjacent or neighbours and this can be denoted by *i~j*. Set $E(G)$ may be defined as

$$E = E(G) = \left\{ (i,j) \mid \left\| \eta_i - \eta_j \right\| < r, i, j \in V, i \neq j \right\} \tag{1.3}$$

$i, j \in \{1, 2, 3, \cdots n\}$. n is number of AUVs present in the formation group. η_i, η_j are the positions of the i^{th} and j^{th} AUVs, r is the maximum possible distance between any two AUVs such that they shall stay within the formation group. This is the sonar range of detection.

A graph G is said to be connected or weakly connected if in between any two vertices *i* and *j*, there exists a path from *i* to *j*. Otherwise, if it is a directed path and there exists a path from each vertex to every other vertex, then it is a strongly connected graph. Based on the graph theory, some scalable formation graphs, i.e. column, line, wedge, and double-platoon formations are shown in Figure 1.4. Each circle in these figures indicates an AUV. The upward dashed arrows are the direction of the motion of the formation structure.

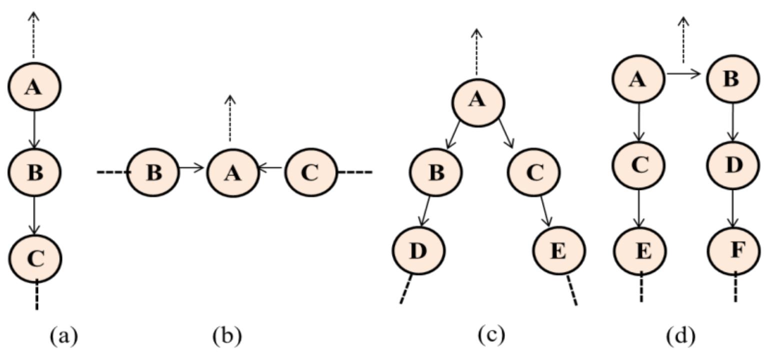


Figure 1.4: Different Types of Formation Shapes: (a) column (b) line (c) wedge (d) double platoon

1.7.2.2 Reference Types

An AUV needs to keep the correct position with respect to the locations of others in the group. Figure 1.5 shows the three reference types for the formation structure. In Figure 1.5

and Figure 1.5, the small circles represent AUVs, and the arrows represent the relation between the AUVs with their references.

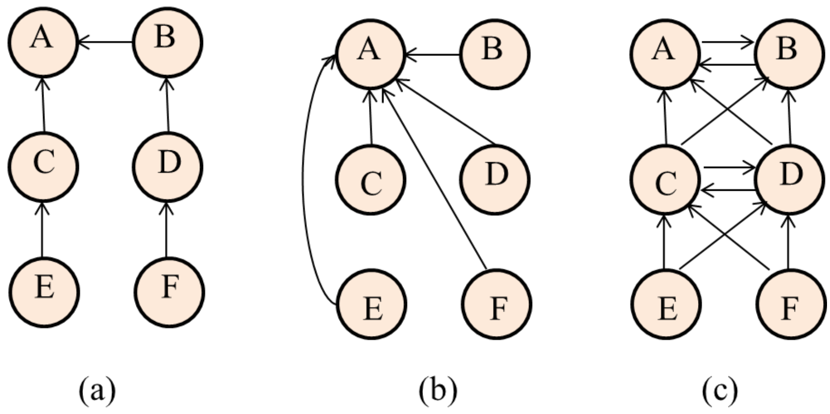


Figure 1.5: Different reference types: (a) predecessor (b) leader (c) neighbour reference types

1.7.2.2.1 Single Reference Type

When a group or subgroups of multiple AUVs follow to a single AUV in the group, it is named as single reference type formation structure. Different reference types are explained below.

1. Predecessor (p) Reference Type

In the predecessor reference type (Figure 1.5 (a)), i^{th} AUV computes its desired position η_i in the formation based on the positions of its predecessor η_j , $i < j$.

For example, in Figure 1.6, the AUV D is the predecessor of the AUV F, and F calculates its desired position based on the location of D.

2. Leader (l) Reference Type

In the leader reference type, i^{th} AUV calculates its desired position η_i based on the position of the leader AUV η^l as shown in Figure 1.5 (b). The most popular approach is the leader-follower approach where some followers maintain formation with a leader and these followers plays the role of local leaders to be followed by other follower's recursively to form a chain or a tree topology. A vehicle computes its correct formation position based on the locations of the leader or local leaders i.e. $\eta_j = f\left(\eta_i, \eta^l\right)$, $i \neq j$, f is a linear function.

3. Neighbour (n) Reference Type

In the neighbour reference type, each AUV computes its desired position η_i based on the position of its neighbour AUVs η_j , $i \neq j$, as shown in Figure 1.5 (c). An AUV with position η_i detects its front, left, and right neighbouring AUVs η_j , $i \neq j$ within a given distance $r_{ij} < r$ (r_{ij} is the distance between i^{th} and j^{th} AUVs), calculates its desired position η_{i_r} according to these detected neighbouring vehicles, and then drives itself to the desired position to stay in formation.

Mathematically $\eta_{i_r} = f\left(\eta_i, \eta_j\right), i \neq j$.

1.7.2.2.2 Hybrid Reference Types

A hybrid reference type is formed by combining two or three single reference types, these single reference types are called as *ancestor reference* types. The four hybrid reference types found in literature are; leader-predecessor (l-p), leader-neighbour (l-n), predecessor-neighbour (p-n), and leader-predecessor-neighbour (l-p-n) reference types.

1.8 Cooperative Coordination Strategies of AUVs

Coordination is the method of establishing stable interlink among vehicles as a result of which, the whole group would be able to solve a complex task efficiently.

The cooperative architecture for multiple AUVs for different control strategies can be extended to include formation feedback [2]. A formation function is used to define a formation error for a class of vehicles so that a constrained motion control problem of multiple systems may be converted into a stabilization problem [39]. There are three cooperative approaches to solve the problems of motion planning and control of a group of AUVs. These are centralized, decentralized and distributed approaches.

1.8.1 Centralized Approach

In centralized approach, a single controller and collision free trajectories are constructed in a workspace. Each AUV is controlled by a central processor control unit as shown in Figure 1.6. All team members know the desired shape, location of formation and desired path to be travelled. The central controller processes all the information in the whole system to achieve the formation control objective.

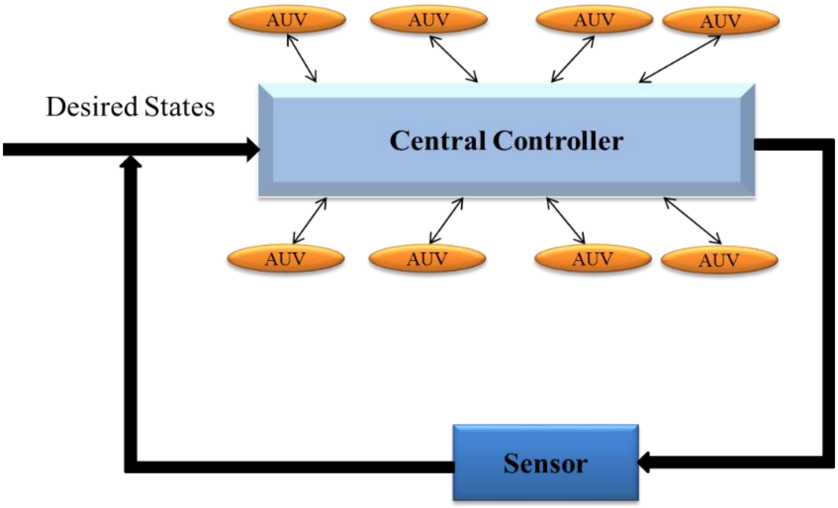


Figure 1.6: Centralized approach coordination of AUVs

Figure 1.6 illustrates the centralized method of coordination of multiple AUVs. Central controller is the central processor that provides input controls to all the AUVs.

1.8.2 Decentralized Approach

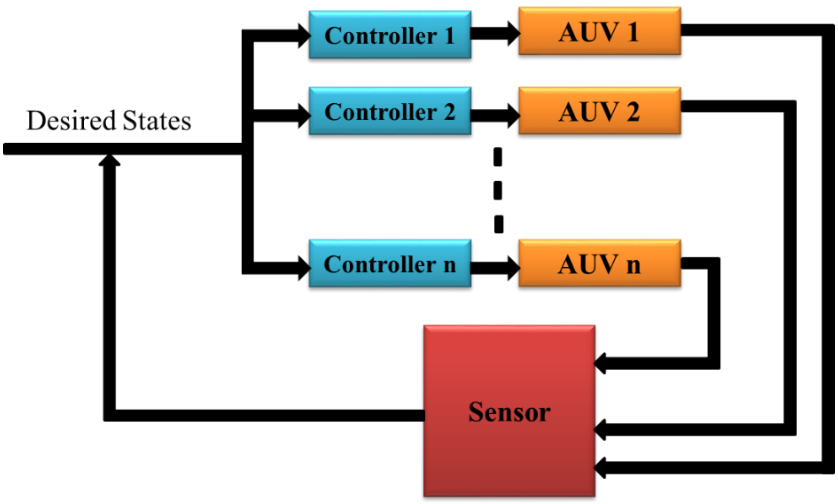
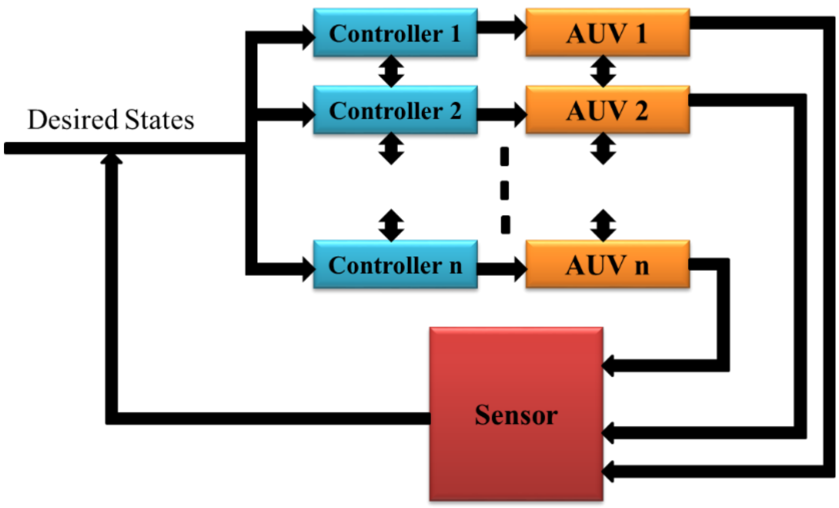


Figure 1.7: Decentralized approach coordination of AUVs

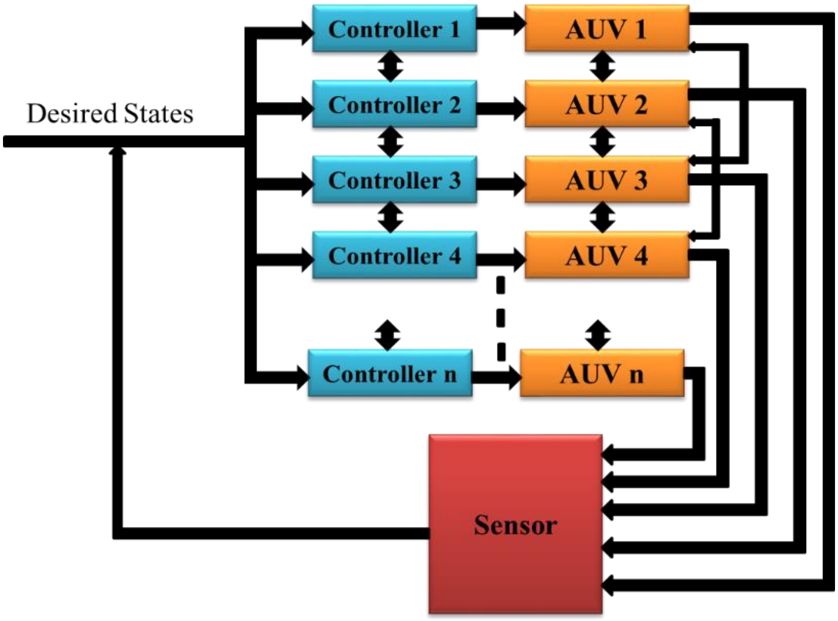
In decentralized control approach, each individual agent of the group is controlled by a separate controller. So individual agent is *autonomous, has decision making power* and *does not depend upon any other agent's decision*. In a team with a large number of vehicles, it needs a huge computational capacity and a large communication bandwidth. In this case it is advisable to use the decentralized coordination strategy. Sometimes for keeping decentralized cooperative control, the agents use the neighbour's information [49]. Here, the decentralized means the control action could be computed in the distributed fashion. The schematic presentation of decentralized coordination structure is shown in Figure 1.7. Figure 1.7 explains the decentralized method of coordination strategy among AUVs. Here each AUV gets input-control provided by a separate controller, but there is no central controller. The communication between neighbours is established to achieve coordination among AUVs. In some special cases, communication between any pair of AUVs can be established to keep coordination, not limited to neighbours.

1.8.3 Distributed Approach

Distributed control involves the use of decentralized, coupled controllers. The control input to any vehicle can be generated by using the information available from its neighbour vehicles. Information must be shared among all vehicles to allow the group as a whole for solving the control problem. Thus, a communication protocol is defined for distributed control. In the distributed control, neighbour AUVs exchange their information with each other via wireless networks [40]. The entire team is modelled as a networked control system.



(a)



(b)

Figure 1.8: (a) and (b) Distributed approach coordination of AUVs

Figure 1.8 (a) illustrates the distributed method of coordination strategy between AUVs. Separate controller provides control input to the individual vehicle. Coordination between controllers as well as between neighbour vehicles is possible. Figure 1.8 (b) is similar to that of Figure 1.8 (a) with the probability of communication among neighbour AUVs.

The advantages and disadvantages of these coordination approaches are mentioned in Table 1.1.

Table 1.1: Advantages and disadvantages of different cooperative coordination strategies

Coordination strategies	Advantages	Disadvantages
Centralized approach	• The centralized formation control is a good strategy for a small team of AUVs, when it is implemented with a single computer and a single sensor to monitor and control the whole team. It gives a complete solution easily.	• The centralized approach has a drawback of computation complexity. The complexity of the algorithm grows with the number of AUVs. • A single AUV has to communicate to the huge number of AUVs. Due to these

		difficulties, it is scalable online. • In order to achieve performance, it requires a massive information flow and high computational power. • This is not robust because of heavy dependence of all AUVs on a single controller.
Decentralized approach	• As separate controllers are used for individual AUVs, this significantly reduces the information flow and the computational power of the controller. Hence it is free from solving complex tasks. Therefore, robustness, flexibility and scalability of the systems increase. • There is no chance of dependency on any central controller whose damage may cause failure of the complete formation system. • It is easy to synthesize and implement this controller. • The human decision can override by this method, if it is necessary. • This is independent of the geometry of the formation. So there is no chance of fault in existing or switching on formation shapes. • The decentralized controller is effective when the communication between the vehicles is limited	• The main problem lies with the decentralized approach is that it is unable or extremely difficult to predict and control the critical points. • Stability margins become worse with increasing vehicle number. • These are very sensitive to external disturbances. • Decentralized controllers are basically model based. • The closed loop system under a controller designed by the decentralized approach has multiple equilibrium points. It is rather difficult to design a controller such that all the equilibrium points except for the desired equilibrium one are unstable/saddle points for a group of many vehicles.

Distributed Approach	• An AUV only transmits the state information to its neighbour's vehicle. • Assuming only one packet for each state transmission, it is possible to avoid the problem caused by multiple packet transmission, in forward and feedback channels formed by sensors and actuators interfaced to the vehicles. • Only neighbour to neighbour communication is required, i.e. communication is light. • Communication is usually asynchronous and locally transmitted, therefore the bandwidth requirement of system is not so high. • It needed light mathematical calculation and easy to synthesize.	• It is very difficult to solve any formation problem by individual AUV in the group, but the problem can be solved by the subgroups present in that group.

1.9 Cooperative Control strategies

In literature, five types of efficient cooperative control strategies are found. These are behavioural approach, leader-follower approach, artificial potential function approach, virtual structure approach and graph-theory approach. There are many papers with little contributions are available presenting other types of formation control strategies. Intentionally the minor approaches are neglected in this chapter.

As most of the flocking controllers deal only with leader-follower and graph theory, this thesis concentrates more on the formation control strategies of AUVs.

1.9.1 Formation Control using Behavioral Approach

In behavior-based control, different desired behaviors are prescribed for each AUV, and the final control law derives from the relative importance of each behavior. The behavioral approach starts from the behavior of individual AUV. The common behaviors are goal seeking, obstacle avoidance, keeping the consistent formation etc. Therefore, more complex motion pattern can be generated by using the individual behavior of separate AUVs. Schematic presentation of behavioral-approach based formation control is shown in Figure 1.9.

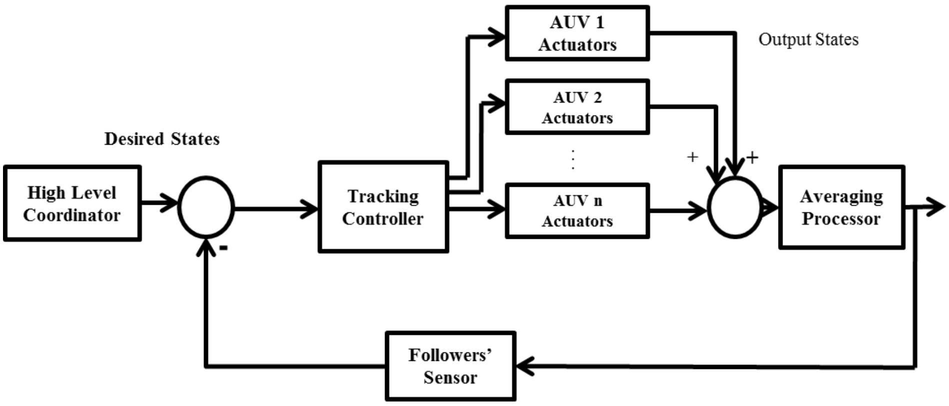


Figure 1.9: Control structure of formation control of multiple vehicles via behavior-based approach and potential field approach

In behavior-based approach, each vehicle has basic motor schema [6]. Each schema generates a vector representing the desired behaviour response to sensory input. Possible motor schemas include collision avoidance, obstacle avoidance etc. as mentioned earlier. The control action of each vehicle is a vector weighted average of the control for each motor schema behaviour.

The architecture of behaviour based formation control of a team of AUVs consists of three levels. These are team behaviour, AUV role and behaviour, and AUV control. Team behaviour comprises the shape of the formation including geometric pattern that the AUV team needs to maintain and reference type that defined the positions of the robots in formation. Robot role and behaviour define the task performed by the individual AUV in formation. AUV control is the control algorithm developed for each AUV within the group.

1.9.2 Formation Control using Leader-Follower Approach

In leader-follower approach, one AUV or more than one AUVs are designated as leaders, which have the global knowledge of the desired paths, while others are designated as followers which have no knowledge of the desired paths. The follower AUVs have to follow the leader AUVs. The schematic presentation of the leader-follower approach is shown in Figure 1.10. In some cases the leader may be a support vessel or a ship [159].

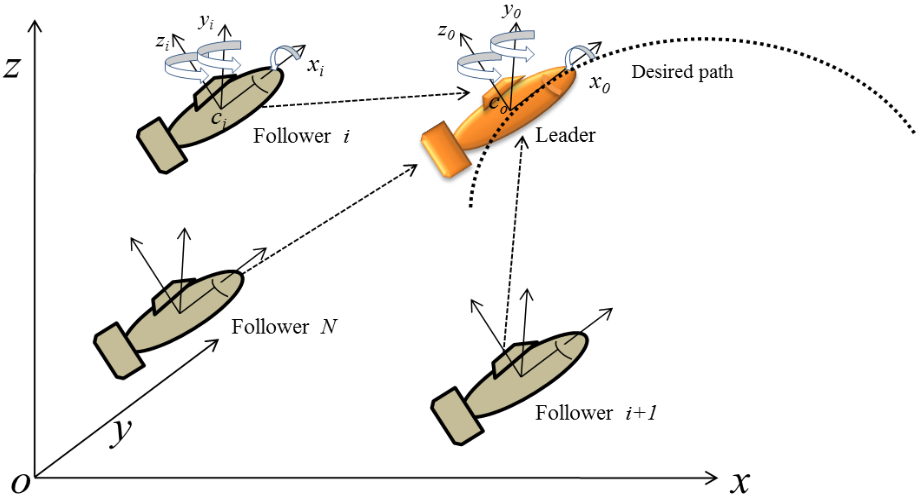


Figure 1.10: Schematic presentation of the leader–follower formation of nonholonomic AUVs

In this approach only local sensor based information is available for each AUV. There are two types of feedback controllers are implemented for achieving leader-follower formation patterns of multiple vehicles [23]. In the first feedback controllers it is assumed that the leaders track is predefined as reference trajectories for the follower AUVs. In the second type of feedback controller it is considered as the follower's track is the transformed versions of the states of their nearest neighbours. The schematic representation of generalized leader follower formation control structure is given in Figure 1.11. But the structure may vary for different situations for fulfilling the particular requirements.

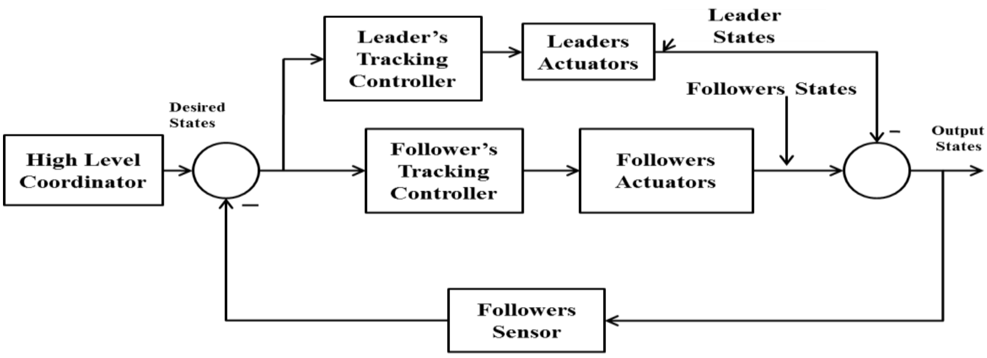


Figure 1.11: Control structure of formation control of multiple AUVs via leader-follower approach

Starting from arbitrary points in space, the leader AUVs are expected to track the desired trajectory maintaining formation .i.e. the errors found between the positions of desired trajectory and positions of leader AUV approach to zero as follows.

$$\lim_{t\to\infty} \left| \eta_l - \eta_d \right| = 0 \tag{1.4}$$

where

η_l = position and orientation vector of leader AUV

η_d = position and orientation vector of the desired trajectory

Throughout movement period, it is necessary that the distance between leader and follower AUVs remains constant.

$$\lim_{t\to\infty} \left| \eta_l - \eta_f^i \right| = d \tag{1.5}$$

where η_f^i is the position vector of i^{th} follower AUV. d is the desired distance between leader vehicle and i^{th} follower AUV.

Some techniques of formation control of multiple AUVs through leader-follower approach are briefly explained below.

1.9.2.1 Distance-Angle leader-follower (DALF) and Distance-Distance leader-follower (DDLF) Method

One of the most popular methods of leader-follower formation control is computing the position of follower AUVs by *distance-angle* and *distance-distance* formula. The position of the first follower which is neighbour to the leader AUV can be computed by taking a Euclidean distance from the centre of mass of the leader agent with making certain predefined desired Euclidean angle. This is the *distance-angle* or L-ψ method of finding the position of first follower.

Mathematically

$$\eta_1^f = f\left(l_{12}, \psi_{12}\right) \tag{1.6}$$

where η_1^f is the position of the first follower AUV, f is a linear function, l_{12} and ψ_{12} are the distance and angle between the leader and the first follower AUV respectively.

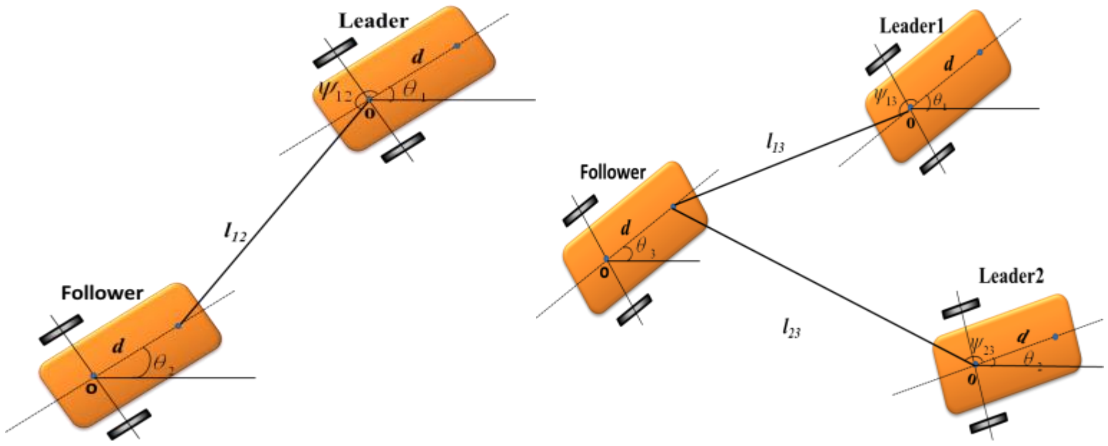
The schematic presentation of L-ψ method is shown in

Figure 1.12 (a). The position of the second follower with respect to the leader, which is the neighbour of the first follower can be computed by using the Euclidean distance from both the leader as well as the first follower. This is the distance-*distance* or $L-L$ method of finding the position of second follower or ordinary follower.

It can be presented as,

$$\eta_2^f = f\left(l_{12}, l_{23}\right) \tag{1.7}$$

where η_2^f is the position of the second follower AUV, l_{23} is the distance between the first follower and second follower AUVs.



(a) (b)

Figure 1.12: (a) Three AUVs in formation (b) formation

The schematic presentation is shown in Figure 1.12 (b). The distance-angle method of control is like a ring of a chain consists of a leader and a follower AUV only. But in case of distance-distance model, the AUVs may stay in different shapes such as column, line, triangle, wedge diamond etc. One AUV follows another two AUVs. L-L and L-ψ controller may be developed for multiple vehicles using internal stable dynamics [20], [54], [78], [124], [172], [184]. The leader follower approach may be implemented using distance and approaching angles [77].

1.9.2.2 Virtual Leader Method

Another technique of leader-follower formation control strategy is the use of virtual leader in the group. In virtual leader concept, a virtual vehicle follows the desired path and the other vehicles follow the virtual vehicle [12], [24], [35], [115], [128], [149], [162]. All other AUVs will move in a parallel manner with the orientation of the virtual leader [167], [181].

1.9.2.3 Line of Sight (LOS) and Vision-based Method

Line of sight (LOS) guidance method is used to control multiple AUVs to follow the desired trajectory LOS method may be used for developing crosstalk control law of formation of multiple surface vessels along with controlling the forward speed. Control of multiple vehicles is carried out by using vision-based localization method [75], [103], [107], [183], [187]. Very often, localization of multiple vehicles accomplished by extended Kalman Filter (EKF) [107], [116]. LOS method of leader follower control strategy in the presence of ocean current disturbance is utilized to solve a formation task in [132].

1.9.2.4 Consensus Method

In consensus method, the connectivity among the AUVs is checked by considering the whole system to be consisted of many local groups. Formation is achieved by communicating all the local groups as a whole. Decentralized neighbour to neighbour local communicated controller may be developed to keep the formation of vehicles [33], [65]. In this method more than one leader vehicles drive the formation structure. In some cases position of virtual leader AUV may be used to build the consensus protocol [81]. Consensus tracking control may be used for formation control of multiple AUVs [84]. Consensus tracking method may be applied to the individual AUV and then applied to the whole formation group [152]. Leader-follower formation control of multiple AUVs based on finite time consensus algorithm is explained in [157].

1.9.2.5 Other Techniques of Leader-Follower Control Strategy may be applied to Formation Control of AUVs

Leader-follower controller may be developed using the information available from on-board sensors of the mobile agents [91], using relative motion theory [87], [111], using bond-graph modelling framework [74] or using steady state error analysis [55], parametric uncertainties [17], graphical inequalities [34] or using both parametric uncertainties and external disturbances [46]. Leader-follower approach with many co-leaders may be taken into account when controlling multiple UAVs [85]. The formulation may be approached through minimally persistence co-leader structure [110], through pre-existing diagraph existing among leader vehicles as well follower vehicles [123], [126] where there are two leaders are considered for keeping rigid formation without knowing information about the reference velocity. A formation is called a minimally persistent if the graph containing all the information associated with this is persistent, and the graph is minimally persistent if it is constraint consistent and minimally rigid. Leader-follower formation control may be considered as decentralized master follower formation control of multiple [106]. Any front acts as a master agent. The master agent only broadcasts the status of itself. Other agents decide their own status based on the status of the master agent as well as consensus on board data. Formation control of multiple AUVs deals with the mooring control in chains form can be developed using the leader - follower method [23].

Leader-follower formation control of multiple spacecraft is achieved using output feedback controller through cellular spacecraft orbits [58], through artificial potential functions associated with follower spacecraft [59], [63] or using relative position and velocity of the spacecraft [66].

By using the virtual concept of leader-follower formation approach the objects can transport from one place to another [118]. Here the object to be transported is considered as a virtual leader vehicle and the vehicles which transport the object are considered as follower vehicles. A fork-lift type constraint may be used to control the relative orientation between the leader and each follower vehicle. This constraint is also used to generate the path to be travelled by the system. Formation control of multiple agents with time varying formation pattern may be considered [70]. Here the concept of virtual leader along with graph theory for communication topology is used for keeping formation among agents. Using virtual leader concept, the controller may be developed on the basis of flatted property of individual vehicle which splits from the group to avoid collision and again can join in the group for keeping formation in the time parameterized path [35].

1.9.3 Formation Control using Virtual Structure Approach

The concept of virtual structure was first introduced in [37]. The entire formation group is considered as a rigid body (Figure 1.14). Control methods are developed to force a group of AUVs to remain in a rigid formation. When the structure moves, it traces out desired trajectories for each AUV in the group. In virtual structure approach, the controller is derived in three steps. First, the desired dynamics of the virtual structure are defined. Second, the desired motion of the virtual structure is translated into desired tasks for each agent. Finally, the individual tracking controller for each AUV is derived. The generalized formation control structure is shown in Figure 1.13.

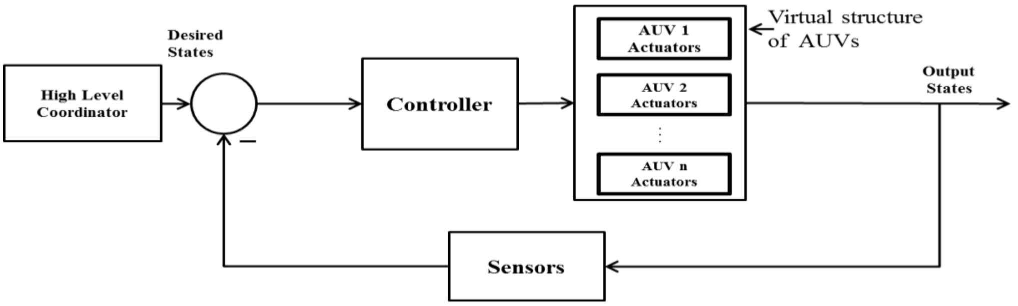


Figure 1.13: Control structure of formation control of multiple AUVs via virtual structure approach

The formation of *n* number of AUVs can be accomplished through virtual structure approach if the following condition satisfies.

$$(\eta_c) - (\eta_d) = 0 \tag{1.8}$$

where η_c is the position vector of *N* AUVs in formation structure. η_d is the position vector of desired trajectories.

$$\eta_c = \frac{1}{n}\sum_{i=1}^{n}\eta_i \ , \ i = 1, 2, 3, \ \dots, \ n \tag{1.9}$$

$$\eta_i = [x_i, y_i, \psi_i]^T \ , \ \eta_d = [x_d, y_d, \psi_d]^T \tag{1.10}$$

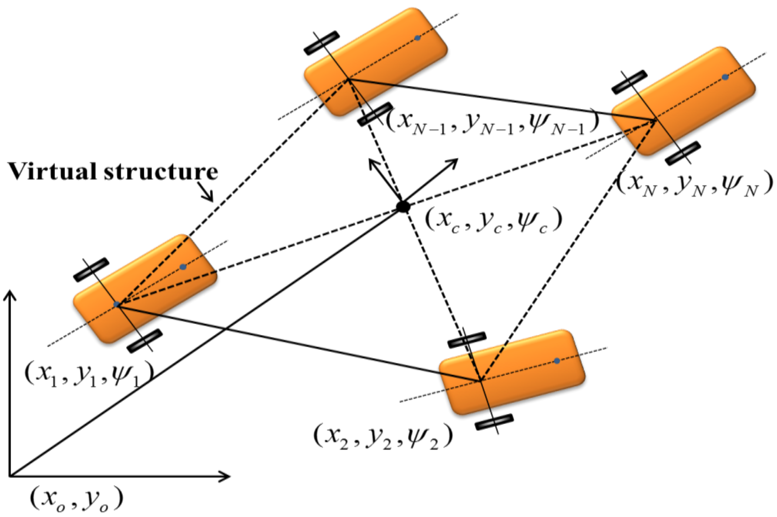


Figure 1.14: Geometric relationships among AUVs and the imposed virtual structure

Some techniques of virtual structure approach are presented here. Commonly the virtual structure approach is usually used in spacecraft or small satellite formation flying control [38]. But in the modern era this technique applies in the field of AUV formation.

Formation control of multiple AUVs may be accomplished using hierarchical virtual structure [120]. In this method the whole team is divided into several groups. Individual group is behaving as a distinct virtual structure.

Other virtual formation techniques that can be applied to the formation control of AUVs are as follows. When a virtual structure concept is used for formation control of multiple spacecraft, the dynamics of the spacecraft may be restricted to a three body problem [72]. The centre of the virtual body assumes to follow a nominal trajectory around the liberal point. The controller is developed to keep all the vehicles at equal distances from the centre of the virtual body. The relative formation of the vehicles is maintained by using a technique called $\theta - D$ methothe distance bearing techniqueistance bearing technique. Formation control of multiple mobile robots may be achieved considering the combination of dynamic surface design technique and the virtual structure approach [125]. Formation control of multiple aircrafts through virtual structure can be achieved using a formation feedback method [38], [2]. Formation feedback method is the subsuming of the leader-follower, behavioral and virtual structure approach which increases the robustness of the system. The coulomb virtual structure approach may be used for formation control of multiple vehicles [67]. Here Columb force plays the major role for clustering of aircraft to achieve a fixed configuration. The virtual structure method can combine with leader-follower and behavioural approaches to obtain formation control of multiple vehicles [2].

1.9.4 Formation Control using Artificial Potential Function Approach

Artificial potential is defined as the interaction control forces between neighbouring AUVs and are designed to maintain a desired inter AUV spacing. There are two types of artificial potential functions are considered. These are attractive potential function (APF) and repulsive potential function (RPF). Generally APF is used to calculate the potential field between AUVs and the desired trajectory positions also between the AUVs themselves to keep them within the group when the distance between them becomes more than a reasonable distance. But the RPF is always calculated between the AUVs themselves as well as the vehicles and obstacles when the distances between them become less than a safety distance to avoid collision.

1.9.4.1 Attractive Potential function (APF)

To avoid group splitting, there should an APF exist between the AUVs. This is an inter AUV distance (r_{ij}) dependent function between i^{th} and j^{th} AUVs. This value should be zero when $\eta_i = \eta_j$ and possess maximum value when $\left\|\eta_i - \eta_j\right\| \geq d$. d is the safety distance. Generally the choice of APF between AUVs is the standard parabolic which grows quadratically with the distance between the AUVs. The value of APF may be taken as

$$U_{att}(\eta_i,\eta_j) = \frac{1}{2}k_{att}r_{ij}^{\ 2} \tag{1.11}$$

where $r_{ij} = \left\|\eta_i - \eta_j\right\|$ is the Euclidean distance between the position of i^{th} and j^{th} AUVs. k_{att} is a positive scaling factor. The gradient (∇) of this function which is a vector quantity may be represented as

$$\nabla U_{att}(\eta_i,\eta_i) = k_{att}\left(\eta_i - \eta_j\right) \tag{1.12}$$

The force of attraction between the AUVs is the negative gradient of attractive potential between them and is presented as,

$$F_{att} = -\nabla U_{att}(\eta_i, \eta_j) = -k_{att}\left(\eta_i - \eta_j\right) \tag{1.13}$$

Similarly the potential function between AUV position η_i and the coordinated position of the desired trajectory η_d can be represented as

$$F_{att}^{d} = -\nabla U_{att}^{d}(\eta_i, \eta_d) = -k_{att}^{d}\left(\eta_i - \eta_d\right) \tag{1.14}$$

k_{att}^{d} is the positive constant which presents a scaling factor.

1.9.4.2 Repulsive Potential Function (RPF)

To avoid collision among AUVs there is an RPF should exist within the group. This function possesses maximum value when there is a chance of collision between AUVs i.e. when $\eta_i = \eta_j$ and asymptotically converges to zero when $\left\|\eta_i - \eta_j\right\| \geq d$. So the AUVs would be able to maintain a certain minimum safety distance between them. Considering the linear nature of the problem the RPF between i^{th} and j^{th} AUVs is given as $U_{rep}(\eta_i, \eta_j)$. This potential for a group of n AUVs may be given as $\sum_{i=1}^{n}\sum_{j=1}^{n} U_{rep}(\eta_i, \eta_j), i \neq j$

However, the repulsive potential increases gradually when the AUVs are approaching near to each other and decreases gradually when they are diverting from each other. At a certain minimum distance the repulsive potential becomes zero. The negative gradient of the RPF field is given by

$$F_{rep} = -\nabla \sum_{i=1}^{n}\sum_{j=1}^{n} U_{rep}(\eta_i, \eta_j), i \neq j \tag{1.15}$$

The total potential function of the whole system is the summation of the attractive as well as repulsive potentials.

The negative gradient of the potential function may be presented as

$$F = F_{att} + F_{att}^{d} + F_{rep} \tag{1.16}$$

Using these potential equations the formation control law is developed according to the requirements.

Some techniques of formation control using artificial potential functions that can be applied to the formation control of multiple AUVs are explained below.

Virtual leaders can be used to AUV and manipulator group geometry and direct the motion of the group [41], [169], [176]. Virtual leader is a moving reference point that influences the vehicles in its neighbourhood by means of artificial potentials. The potential energy is calculated based on the distance between the two vehicles or vehicle and obstacle. Collision between vehicles can be avoided using artificial potential function [51], [52], [127]. The potential becomes infinite when approaching collision and decreases gradually as the distance between them increases. The RPF as well as APF are calculated based on the distance between vehicles, vehicle and goal position and vehicle and obstacle [43], [104].

APF based formation controller for controlling multiple mobile robots can be developed by using *p*-type bump function [80], communication delay [100], consensus protocol [119] or fuzzy rule based swarm optimization [131]. Swarm of multiple unmanned vehicles into formation is possible using the artificial potential function and limiting functions [98]. Parameters for swarming are chosen based on desired formation and user defined constraints. This approach is computationally efficient and well scalable according to different swarm sizes.

1.9.5 Formation Control using Graph-Theory Approach

Formation control of AUVs through graph theory approach is very popular. In this technique, the AUVs exchange information according to a pre-specified communication digraph (directed graph) [42]. Each AUV is provided with information only from a subset of the group. A specific subset of a set of neighbour vehicles is presented in a communicative digraph (Figure 1.15). Different techniques of formation control of multiple AUVs through graph-theory approach are explained below. Sometimes graph-theory based fixed topology are used for formation control of multiple AUVs [163], [179].

Consider a vector as;

$$h = h_p \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} \in \mathbb{R}^{2n} \tag{1.17}$$

where $\otimes$ denotes the Kronecker product. If there are vectors $q, w \in \mathbb{R}^{2n}$ such that $(x_p)_i(t) - (h_p)_i = q(t)$ and $(x_v)_i(t) = w(t)$, for *i = 1 … n*, then there are *N* number of AUVs are in formation h at time *t.*

where $x_p = ((x_p)_1, ..., (x_p)_n)^T$ is the vector of position like variable. $x_v = ((x_v)_1, ..., (x_v)_n)^T$ is the vector of velocity like variable.

AUVs converge to formation h if there exist $\mathbb{R}^n$ valued functions $q(\cdot), w(\cdot)$ such that $(x_p)_i(t) - (h_p)_i - q(t) \to 0$ and $(x_v)_i(t) - w(t) \to 0$ as $t \to \infty$, for *i = 1 … n.*

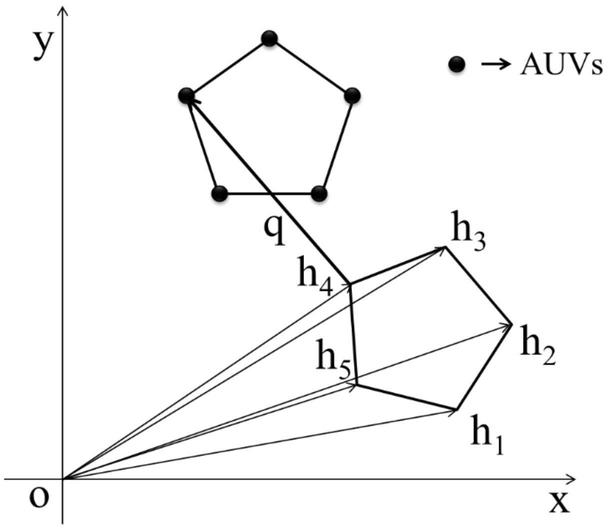


Figure 1.15: AUVs in formation based on graph theory [42]

Figure 1.15 shows the interpretation of the vectors connecting AUVs. Two types of control structures exist for establishing communication between a pair of AUVs. These are symmetric control and asymmetric control [71]. In case of symmetric control, for keeping the distance between two conjugate vehicles, a joint effort is provided from both the AUVs. For this reason the positions of AUVs as well as distance between AUVs maintain a feasible and active value. As a result the edges of graph between those two AUVs become undirected edge. If there is a numerous pair of AUVs, then they maintain a distance and position. So the formation becomes *rigid*. But in case of asymmetric control structure, out of a pair of AUVs, one AUV actively maintains the position and distance between them by receiving the information broadcasted from the second one. The first vehicle also senses the position of the second vehicle and takes the decision of its own. More or less due to this reason, in case of asymmetric control the communication complexity in terms of information sensation for the formation control is reduced to half. The graph forming asymmetrical control structure is a directed graph and the edge connecting the pair of vehicles directed from first to second vehicle. Formation methods using graph theory approach are explained below.

Formation control of multiple under actuated AUVs may be achieved using relative information between the vehicles [25], consensus protocol among vehicles [112], communication delay taking into account of the system of vehicles [89], [94] passivity properties of dynamic system considering dynamic uncertainties of the system [44], using geodesic equations which are derived in suitable coordinate system [82] etc.. The control of multiple vehicles can be achieved using persistence and rigidity of the formation graph [71], [83], [86]. Formation control problem of multiple mobile robots may be solved through the game theory approach [92]. Here particularly Nash differential game approach along with finite horizon cost function through the open-loop is used to solve the problem. Formation control of multiple spacecraft based on graph theory approach can be achieved using fixed and switching topology [69]. These techniques can be applied to the formation control of multiple AUVs.

The advantages and disadvantages of all the control strategies are presented in Table 1.2.

Table 1.2: Advantages and disadvantages of different control strategies

Control Strategies	Advantages	Disadvantages
Behavioral Approach	• It is natural to derive control strategies when AUVs have multiple competing objectives. • An explicit feedback is included through communication between neighbours. • Behavior-based approach is decentralized and it may be implemented with significantly less communication.	• The group behavior cannot be defined explicitly. It is difficult to analyse the approach mathematically and obviously the group stability as it possesses high complexity in mathematical computation. • To reduce the computational complexities upto some extent, it is necessary to reduce the

	• Since it is a decentralized approach, it is suitable for a large group of AUVs. • By using this method a group of AUVs can be guided in an unknown and uncertain environment.	dynamics of individual AUV to single integrator dynamics. • Kinematic model of the AUV is more complex, limiting the applicability of this approach in practice. • The convergence of a group of AUVs to a desired configuration is difficult.
Leader-Follower Approach	• An advantage of the leader-following approach is that it is relatively easy to understand and implement. • The formation can still be maintained even if the leader is perturbed by some disturbances. • The group objective is a path following problem, which is performed by the leader AUV only, so this problem would not be a task for the following AUVs. • Sometimes the leader-follower approach reduces the total huge task to individual small tasking problems via the states of leader AUV, so that the followers should know the states of the leader AUVs only.	• There is no explicit feedback to the formation, that is, no feedback from the followers to the leader in this case. • If the follower is perturbed, the formation cannot be maintained, which shows the lack of robustness of this type of approach. • The leader is a single point of failure for the formation. • Considering individual leader-follower pair of agent, it obeys local optimal formation control law. But connecting all individual optimal formation control law does not guarantee that it is a global optimal controller.
	• It possesses certain robustness towards perturbation of any vehicle from the group.	• Requiring the formation to act as a virtual structure limits the class of potential applications

Virtual structure Approach	• It is fairly easy to prescribe the coordinated behaviour of the group, and the formation can be maintained during the manoeuvres, i.e. the virtual structure can evolve as a whole in a given direction with some given orientation and keeping a rigid geometric relationship among multiple AUVs.	such as when the formation shape is time-varying or needs to be frequently reconfigured, this approach may not be the optimal choice. • The virtual structure and leader-following approaches require the full state of the leader or virtual structure be communicated to each member of the formation.
Artificial potentials Approach	• There is no need to ordering the AUVs as there is no leader is considered. Any vehicle can interchange with any other AUV. • There is no much harm is case of missing of an AUV.	• When the repulsive potentials among the AUVs and the vehicle and obstacle are considered, the potential functions are calculated based on the local minima. So when the number of obstacles increases in the environment, the number of local minima increases instead of steering towards global minima. So the formation structure is not stabilized. • Another major disadvantage of this method is that, the total potential function of a system is the summation of the attractive as well as the RPFs. Sometimes there are equilibriums may appear by summation of these two potential functions, where

		the composite vector AUVs and the system may trap in undesired equilibrium points. The calculation of these equilibriums and the trajectories which are not converged to the desired formation is very difficult.
Graph-theory Approach	• Formation controller is easily developed only using some basic geometrical methods without using rigorous mathematical calculations.	• Assuming the positions of vehicles as the vertices of the graph, the graph should be connected otherwise it is very difficult to develop the control law.

1.9.6 Other Existing Control Strategies

Other than major formation control strategies explained, there are many different techniques of formation control are found in literature. Some of these techniques are briefly explained here which may be applied to the formation control of AUVs.

1.9.6.1 Geometrical Formation Control

A group of AUVs may be assigned to manoeuvre and reach at a particular position in a collision free manner through geometrical coordinated path following (CPF) formation control problem [22], [108], [113], [150], [153], [170], [171], [178] along with communication topology taking into account [24]. The CPF problem of multiple AUVs may be developed using leader-follower approach [164], and in distributed manner [166]. CPF principle maybe applied for autonomous surface vehicles with discrete-time periodic communication [26] with the bidirectional communication taking into account [27] for a group of underactuated AUVs with communication failure and time delay condition in [29]. Geometric approach of formation control may be achieved using consensus algorithms with provable convergence [113], velocity optimization technique [122] and using Jacobi transform of the horizontal kinematics of the AUV [105]. Geometric formation control of multiple AUVs may be possible by utilizing Jacobi shape theory [165]. The formation control of multiple AUVs based on the path following control of single AUV is presented in [152], [156], [160], [168], and [180]. To solve the coordinated path following problem, hybrid controller maybe developed [117]. The controller is called hybrid, because the vehicles are controlled by two controllers depending upon their orientation. If the orientation is within a particular value, then the vehicle is driven by one control algorithm otherwise by another control algorithm. The orientation and translational control are decoupled and geometric formation controller for multiple AUVs are developed in [158].

Flying control of multiple spacecraft may be achieved using lumped state errors [28]. The lumped state error is the submission of two types of errors such as, attitude/position error and formation keeping errors. Quaternion-based attitude-coordination of multiple spacecraft may be achieved considering input saturation with separation principle and robust control technique [28]. The control of multiple robots may be carried out by coalition method [56]. If the whole team is divided into different subgroups and individual subgroup is assigned to complete individual tasks then it nominated as the *coalition*. The task allocation to each individual subgroup is accomplished using modified Shehory and Kraus' Algorithm. Then multi robot formation of all coalitions is done through iteration process. These formation techniques can be applicable to the case of multiple AUVs.

1.9.6.2 Formation Reference Point (FRP) Method

In some formation control problems, a virtual formation reference point (FRP) of formation structure assigned to track a predefined path [18]. Here the complete tracking problem is divided into two sub problems, i.e. a geometric task and dynamic task [31]. The geometric part guarantees the FRP and hence the formation takes the path, i.e. FRP follows the predefined path. The dynamic part confirms the accurate speed control of vehicles along the path.

1.9.6.3 Consensus based Formation Control Method

Formation control may be achieved through second order consensus based protocol for multiple AUVs [73] or multiple spacecraft [61]. Formation control of multiple agents may be achieved using intermittent information exchange [76].

1.9.6.4 Other Methods

A simple PD and PID based controller may be developed to solve the difficulties of complex formation of multiple AUVs taking the gravity uncertainty into account [51]. The synchronization problem of Kuramoto oscillator networks with communication topology and nonzero speed and time delay may be considered for formation control of multiple agents with linear continuous time dynamics [30].

A decentralized formation controller may be developed using vehicle speed profile of individual vehicles [79] or Lagrangian mechanics of constraint functions such as distance between members, position constraints, combined constraints, formation average position and formation variance [19]. Decentralized controller for Platoon formation is possible using average positions and configurations of the vehicles [50] or this may achieved by task priority inverse kinematics [48].

Distributed control scheme for multiple mobile robots with parameter uncertainty and actuator saturation may be developed based on horizon H^{∞} control scheme [53]. In this scheme, the whole multi robot system is divided into different cooperative subgroups. These subgroups communicate with each other for keeping formation stabilization as well as for performing assigned tasks. This also reduces the computational complexity.

In permutation-invariant, formation control system may be considered as a set of robot configurations without assigning tasks to individual robot [60]. Here the formation is presented by the invariant coefficients of complex polynomials whose roots are presenting the robot configurations. These polynomials provide an effective permutation-invariant formation. Robots became aware about their tasks after reaching at the eventual goal.

The cluster space state method may be used for achieving formation control of mobile robots [96]. In this method the whole team of vehicles is divided into different clusters of limited sizes. Local controller is designed for each cluster, and then formation control is carried out by centralized method [111].

A vision based parallel and circular formation control of multiple non-holonomic robots may be developed using consensus control law which is distributed in nature [99], by coupling of individual agent coordinates in one dimension [101], based on expansion and contraction paradigm [102] or using the properties and area of representation space (RS) [47].

The space representation is constructed to describe different attributes of the team of mobile robots. The researchable area in the RS indicates the ability to accomplish the task by providing the external and internal constraints of the system. Dynamic task assignment and path planning for multiple AUVs using self-organizing map (SOM) neural network and novel velocity synthesis approach is explained in [151]. Range-only based formation control of multiple AUVs are expressed in [155].

Stable periodic formation control of multiple non-holonomic vehicles may be achieved by arranging the vehicles in a cyclic interconnection topology known as 'cyclic pursuit' in mathematics (Figure 1.16) [45]. This may possibly by a fixed circular formation [173], as well as centre is fixed with various radii [174]. Formation control of multiple spacecraft can be developed using hierarchical genetic algorithm [114].

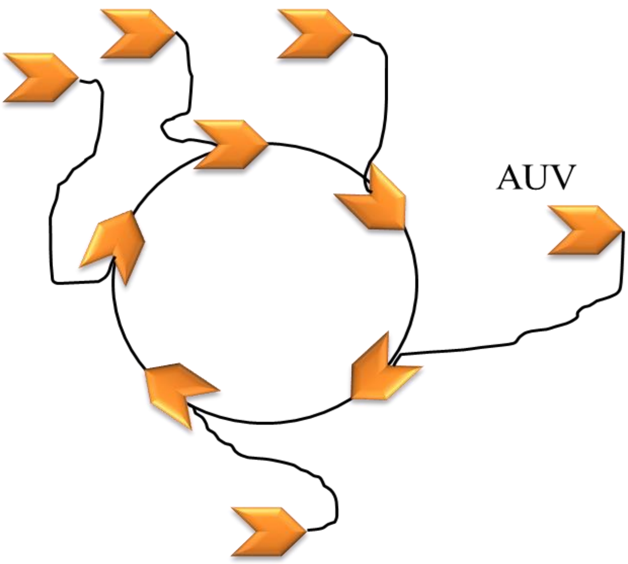


Figure 1.16: Cyclic-pursuit of multiple AUVs

1.10 Cooperative Control Sub-problems

Apart from the control structures, there are several sub-problems accompanied with cooperative control of multiple AUVs. These are collision and obstacle avoidance, cooperative shape generation, switching between shapes according to situation, path generation, path following and trajectory tracking. These are briefly explained below.

1.10.1 Collision and Obstacle Avoidance

When a group of multiple AUVs move in formation, it is necessary to avoid collision between themselves as well as to avoid collision with the solid obstacles intersecting the coordination path to be travelled with the group. Obstacle avoidance is highly essential sub-problem in cooperative control. The obstacle may be static or dynamic [122]. In case of a moving obstacle, the obstacle may appear suddenly which are beyond the prior knowledge of the AUVs.

For avoiding collision between vehicles there should exist a repulsive force between them. Similarly, there is a repulsive force should be established between the vehicle and the obstacle such that there is no collision happens with the solid obstacle appears on the desired path. AUVs should avoid collision with the obstacle and should keep a safe distance from the obstacle. Mathematically, it is denoted as;

$$\lim_{t \to \infty} \left\| \eta_i - \eta_j \right\| = 0 \tag{1.18}$$

$$\lim_{t \to \infty} \left\| \eta - \eta_{obs} \right\| = d_s \tag{1.19}$$

$\eta_{obs} = [x_{obs}, y_{obs}, \psi_{obs}]^T$ is the position coordinates of the obstacle, d_s is the safest distance of the AUV form obstacle.

One of the best methods for avoiding collision between vehicles as well as a vehicle and obstacle is based on artificial potential function. In this method an RPF between AUV and obstacle is developed which is inversely proportional to the norm of the distance between them. So as the distance between the AUV and the obstacle decreases the repulsive force increases and is infinite at zero distance [21]. To design the obstacle avoidance algorithm, the area consisting obstacles is separated into different regions. The manoeuvring area is divided into three different parts such as: safety area, avoidance area and danger area. The total potential energy is divided into two different parts. One is potential energy between the two AUVs, and another is potential energy between the vehicle and the target.

Another important method is using the Fuzzy logic method. Mixed Integer Quadratic Programming (MIQP) optimization method may be used to avoid collisions between the vehicles [69], [185]. Here MIQP method is used to detect the obstacles in the environment and Fuzzy logic controller is used to avoid the obstacles.

Other Methods of Obstacle avoidance

A method based on the limit cycle process may be employed for avoiding obstacles by generating trajectories of the robot manipulators [15]. Here the shapes of complex obstacles are modelled by unstable limit cycles. The obstacles may be avoided using the same limit cycle method, but in a different fashion in [60]. Here the trajectories of the obstacles are presented as a set of transitional trajectories which are considered as the solutions of the differential equations presenting a stable limit cycle of elliptical shape. These ellipses encircle the obstacles. When an obstacle is detected on the desired path, the new trajectories of the vehicle are generated satisfying that differential equation to avoid the obstacle. When the obstacle is avoided, the vehicle again returns to the original path. By combining this avoidance strategy of individual obstacle, a group of vehicle in formation can avoid obstacles during travelling along the desired trajectories [16].

A dual mode control strategy [14] may be used for obstacle avoidance. The modes are a safe mode where there is no chance of collision with an obstacle and a danger mode arises in an obstacle rich region where there is a chance of collision of a vehicle with another vehicle or with obstacles present in the environment. Collision between vehicles as well as obstacle avoidance is possible using *line-of-sight* obstacle *avoidance* principle [88]. According to this principle, when the leader vehicle feels difficult to see the target point or desired trajectories due to presence of solid obstacle, then this vehicle will move around the obstacle until to see the target point again and move toward the target point.

AUVs can avoid obstacles by using LOS method [62], [186]. When an obstacle comes in LOS of an AUV, a large bounded sphere is considered around the spacecraft. Then translation of this safety sphere from one place to another place with time (Figure 1.17). Collision between the spacecraft can be avoided by using forbidden sphere technique [64]. The spacecraft is bounded by a forbidden sphere and any two spheres will not intersect throughout the manoeuvring period.

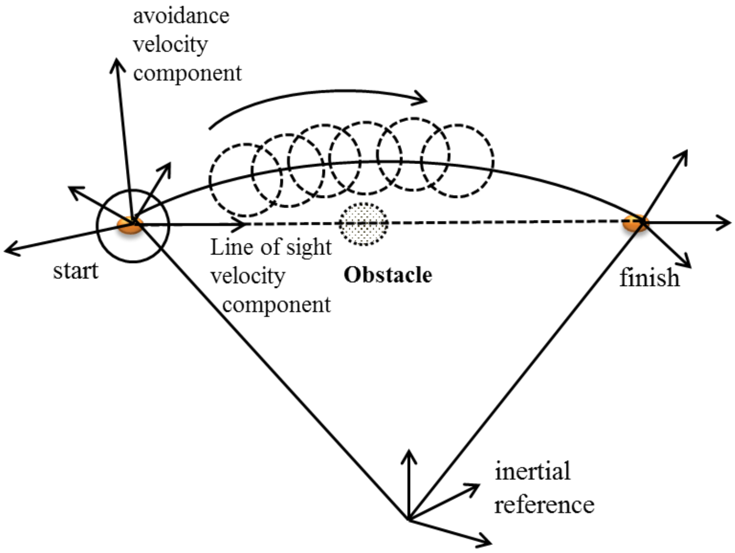


Figure 1.17: Line-of-Sight and collision avoidance path components

The chance of collision between vehicles may be avoided with the use of gyroscopic forces [11]. Static as well as dynamic obstacles are avoided by the multi-vehicle system using a null-space-based behaviour control technique [95]. Sometimes the obstacle may be treated as a virtual leader [111], [124].

1.10.2 Cooperative Shape Generation

Different types of cooperative shapes are described earlier. The shape of the cooperative structure is generated according to the situation of the surrounding environment as well as the communication topology used. Graph theory considering the AUVs as the nodes and the communication vectors as the links plays an important role in the generation of cooperative shapes.

The cooperative control of multiple AUVs may be maintained in different situations of the formation regions separately [93], [182]. The cooperative shapes are generated by a team of AUVs according to required conditions using appropriate mathematical functions. The cooperative shapes may be triangular, ring shape, ellipsoid, rectangular, square, crescent, polygonal etc. The structures of the different shapes of the formations are shown in Figure 1.18.

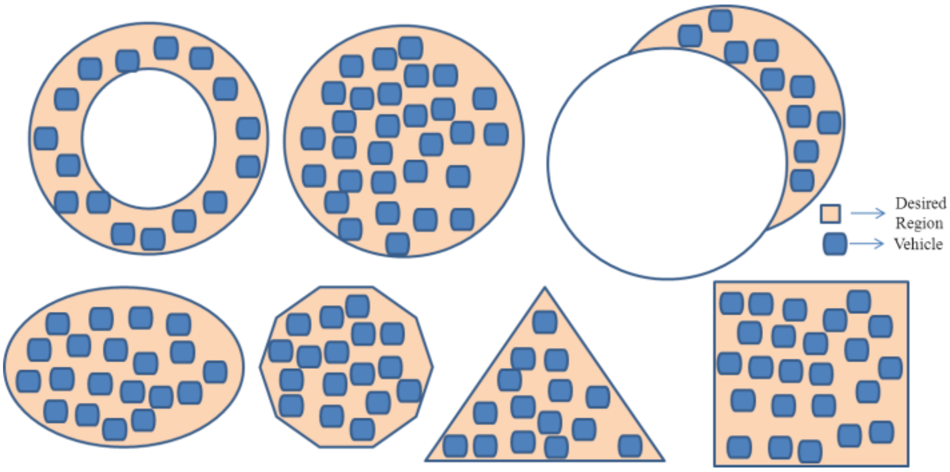


Figure 1.18: Examples of desired regions

Like in [118], the technique of formation control of multiple AUVs may be used in transportation of objects. Here the shape of the formation depends upon the shape of the objects to be transported and the grasping points present the position of the AUVs. Generation of the shape of the formation and change of shape of formation structure is possible easily by applying the imaginary equilateral triangle (IET) method [121]. In this method, any three robots form an equilateral triangle and from that, the shape of the formation structure can be generated. A graph theory based framework for communication topology is used to generate the formation shape is presented in [13], where the importance of minimal rigid graph and graph rigidity is clarified.

1.10.3 Switching Between Cooperative Shapes

If there are no disturbances appear in the cooperative environment, the cooperative shape maintains a rigid shape or the graph rigidity is maintained throughout the motion. But in certain cases it is necessary to change or to split the shape of the coordination according to requirements. It may happen due to change of environmental conditions such as the presence of some uncertainties, entrance of heterogeneous vehicles, presence of solid obstacles, narrowness of formation track, communication failure between AUVs due to attenuation and suppression of strength of communication signals etc.

Cooperative shape may be switched from a straight line to wedge shape for stationary surveillance [90] or may be changed from double platoon to another shape [122]. Figure 1.19

and Figure 1.20 show the schematic representation of cooperative shape change for six AUVs in a group.

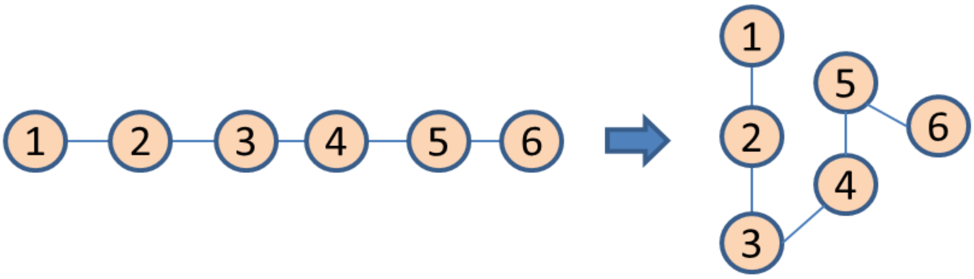


Figure 1.19: Switching from straight line to triangular cooperative shape

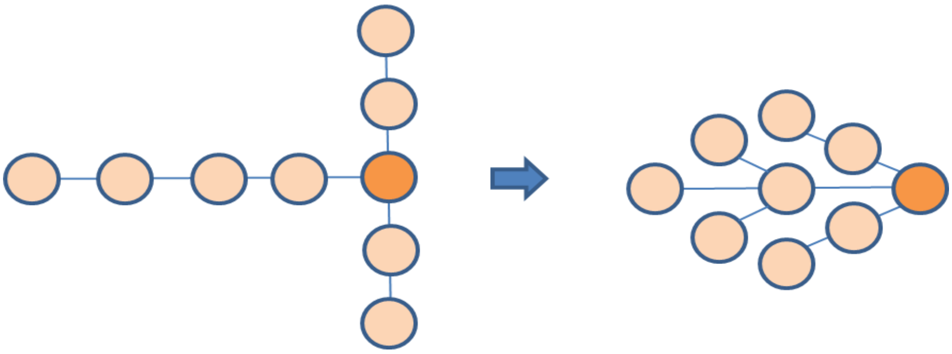


Figure 1.20: Switching from T-formation to diamond formation

By using the phase waves and phase gradient method the shape of the cooperative structure can be adapted in a similar fashion as amoeba in drastic environmental condition [97]. Here the shape of the structure changes according to the present position of the obstacles. When a group of AUVs tries to move into the obstacle rich region, sometimes it needs to pass through a narrow region. So the shape of the coordination changes according the situation. The schematic representation is shown in Figure 1.21. The shape of the formation can be changed and adapted to avoid obstacles by manipulating the potential functions associated with this [43].

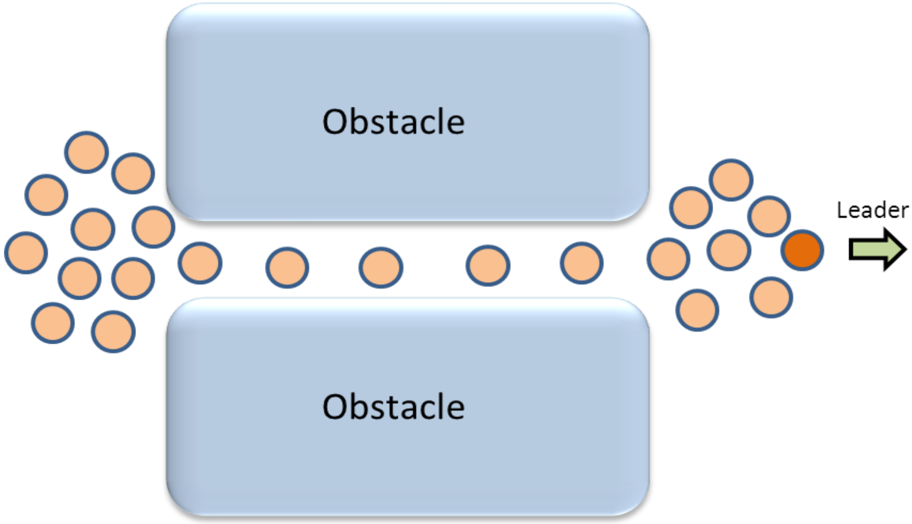


Figure 1.21: A group of AUVs change their formation shape to pass through a narrow region

1.10.4 Cooperative Structure Repair

In case of malfunctions and failure of coordination due to missing or mechanical disturbances of one or many AUVs, the formation structure is disturbed. It is necessary to recover the formation shape again to the fundamental shape. This is possible by mean of ignoring the missing vehicle and re-joining the other vehicles with establishing communication among them [109]. When an AUV gets lost from the group, all other neighbour AUVs break their communication and remake communication with the next nearest neighbour AUVs. The schematic presentation of repairing of cooperative structure in case of missing of an AUV is shown in Figure 1.22. It is possible by repositioning the AUVs in the group with new formation configuration.

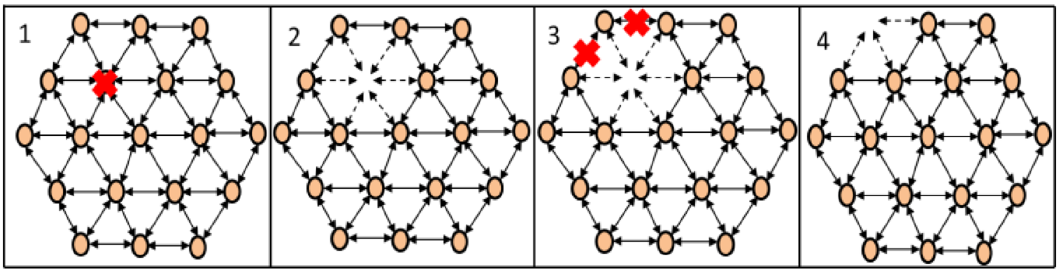


Figure 1.22: Repairing of the cooperative shape in case of missing or failure of an AUV

When an AUV suffers from mechanical failure, it is treated as the loss of the AUV, but this defective vehicle acts as an obstacle for the remaining group of AUVs. So the obstacle avoidance algorithm can be applied here to solve the problem.

1.10.5 Movement of Cooperative Structure

1.10.5.1 Path Generation

The path is generated according to the environmental situations. It depends on the distance and position of the target point where the formation structure to be reached. There are different ways to generate the paths to be travelled by the AUV. The method of path planning or path generation consists of three steps. The first step is the determination of a set of key waypoints (KWPs) on the way towards the goal point. Second step includes the generation of the local constraints of the current state to the next KWP along the planned path. Third step is the optimization of the generated path from the starting point to the goal point by adjusting the KWPs. The waypoints can be generated by using the minimum entropy techniques [154].

Paths are generated using a simple probabilistic random manner (PRM) planner whose local planning algorithms are straight line paths in the plane [57]. Mixed Integer Quadratic Programming (MIQP) optimization method may be used to generate the desired trajectories [69].

1.10.5.2 Path Following

In path following problem, the AUVs had to converge to a pre-assigned desired path [135], [188]. This is specified without temporal laws. Generally in path following problems, the vehicles approach to the desired path smoothly and the control signals are pushed into saturation in less time.

1.10.5.3 Trajectory Tracking

In case of trajectory tracking problems, the AUVs are forced to merge a time parameterized geometric path [135]. The control law designed according to the temporal variations of the desired geometric path. Approaching of the vehicles to the desired trajectory is less smooth than that of the path following problems. Also the convergences of the vehicles to the desired trajectory tracking take more time than that on the path following problems. The degree of difficulty of designing control law for the formation of multiple vehicles is highly dependent on the complexity of the configuration of the AUVs. For fully actuated systems the trajectory tracking problems are clearly understood, but it is not easy for underactuated systems. This is due to more complexity in the configuration of the underactuated systems. So trajectory tracking problems for underactuated systems are more challenging than that of the fully actuated system. As compared to the path following problem, the trajectory tracking approach possesses better performance for carrying control information on time [120].

1.11 Formation Control Stability Notions

Formation control is the control of an interconnected system of multiple AUVs. For safety, robustness as well as for getting the desired performance from an interconnected system, stability analysis of the system is necessary. This is essential to make a system operationally stable. There are three stability notions of formation control. These are string stability, mesh stability and leader-to-formation stability which are interconnected to each other.

1.11.1 String Stability

This type of stability criteria is used to check the stability of the systems operating in platoon structures or hybrid platoon system [133]. This stability method can be applicable to both constrained and unconstrained systems [136]. Parameters require for 'string stability' are position and bearing errors. All the position errors (longitudinal errors) of individual AUVs whether these are changing with time or remain constant and the bearing errors (lateral errors) which are the relative angle between AUVs and its predecessors [137]. String stability presents the uniform bounds of the states of all interconnected systems. [140] is often cited as the first work to prose the string stability for interconnecting systems. The definitions of string stability are explained below.

Define the notations as $\|f_i(\cdot)\|_\infty = \|f_i\|_\infty$ denotes $\sup_{t\geq 0}|f_i(t)|$, and $\|f_i(0)\|_\infty$ denotes $\sup_i |f_i(0)|$ for all $p<\infty$. Similarly $\|f_i(\cdot)\|_p = \|f_i\|_p$ denotes $\left(\int_0^\infty |f_i(t)|^p dt\right)^{\frac{1}{p}}$, and $\|f_i(0)\|_p$ denotes $\left(\sum_1^\infty |f_i(0)|^p\right)^{\frac{1}{p}}$.

The *interconnected* system may be defined as

$$\dot{x}_i = f(x_i, x_{i-1}, ..., x_{i-r+1}) \tag{1.20}$$

where $i \in \mathcal{N}, x_{i-j} \equiv 0 \forall i \le j,\ x \in \mathbb{R}^n,\ f : \underbrace{\mathbb{R}^n \times \cdots \times \mathbb{R}^n}_{r \cdots times} \to \mathbb{R}^n$ and $f(0, \cdots, 0) = 0$.

String stability: The origin .of $x_i = 0,\ i \in \mathcal{N}$ of (1.20) is string stable if for a given $\in > 0$, there exists a $\delta > 0$ such that the following condition is satisfied

$\|x_i(0)\|_\infty < \delta \Rightarrow \sup_i \|x_i(\cdot)\|_\infty < \in$.

Exponentially String stability: The origin $x_i = 0, i \in \mathcal{N}$ of (1.20) is exponentially string stable if it satisfies the condition of string stable and $x_i(t) \to 0$ asymptotically (exponentially) $\forall i \in \mathcal{N}$.

l_p *String stability:* The origin $x_i = 0, i \in \mathcal{N}$ of (1.20) is l_p string stabile if $\forall \varepsilon > 0$, there exist a δ such that

$$\|x_i(0)\|_\infty < \delta \Leftrightarrow \sup_t \left(\sum_1^\infty |x_i(t)|^p \right)^{\frac{1}{p}} < \in \tag{1.21}$$

From the above definitions some special cases are as follows. l_p *String stability* is a special case of *String stability*. Under weak coupling conditions any countable infinite interconnected exponentially nonlinear *string* stable systems are string stable.

String stability is applied to leader-follower formation control of multiple cars like robots in [138]. Here the string stability applied to leader-follower and predecessor-follower coupled vehicles differently and the definition as follows.

Leader-follower string stability: for a step change in $\dot{q}_L$ at any time $t = 0$, the leader-follower interconnected system will asymptotically stable if the following condition is satisfied.

For every $j = 2, \cdots, N_v$, there exists a constant $\alpha_j \in (0,1)$ so that the following closed-loop position error satisfies

$$\max_{t \ge 0} |q_{j,e}(t)| \le \alpha_j \max_{t \ge 0} |q_{1,e}(t)| \tag{1.22}$$

$q_L, \dot{q}_L$ are the geometric state (position and speed respectively) of vehicles.

Predecessor-follower string stability: for a step change in $\dot{q}_L$ at any time $t=0$, the predecessor-follower interconnected system will asymptotically stable if the following condition satisfies. For every $j=2,\cdots,N_v$, there exists a constant $\alpha_j \in (0,1)$ so that the following closed-loop position error satisfies

$$\max_{t\geq 0}\left|q_{j,e}(t)\right| \leq \alpha_j \max_{t\geq 0}\left|q_{(j-1),e}(t)\right| \tag{1.23}$$

The string stability criteria are applied in an array of linear interconnected AUVs under communication constraint conditions in [139].

1.11.2 Mesh Stability

Mesh stability is used for accomplishing stability property of multiple interconnected nonlinear systems. This is a property of the damping disturbance propagation [63]. It has an important role in the attenuation of error in the system which obeys the look-ahead conditions [142], [144].

Look-ahead condition: An interconnected nonlinear system is called look-ahead, if the (*i*, *j*)th subsystem is connected only to the subsystems (*k*, *l*) such that $k \leq i$ and $l \leq j$. The look ahead condition is explained as below.

Consider a system of the form

$$\dot{x}_i = f_i\left(x_i, x_{i-1}, \cdots, x_1\right) \tag{1.24}$$

where $i \in (1,\cdots,N), x_i \in \mathbb{R}^n, f:\mathbb{R}^n \times \cdots \times \mathbb{R}^n \to \mathbb{R}^n$ and $f(0,\cdots,0)=0$.

Exponential Mesh Stability: The origin *x* = *0*, of the dynamical system (1.24) is globally exponentially mesh stable if the following conditions are satisfied.

(i) given $\in > 0$, there exists a $\delta > 0$ such that

i. $\left\|x(0)\right\|_\infty < \delta \Rightarrow \left\|x(t)\right\|_\infty < \in$

(ii) $x \to 0$, exponentially $\forall x \in \mathbb{R}^n$ (1.25)

(iii) $\left\|x_i(t)\right\|_\infty \leq \left\|x_i(t)\right\|_\infty^{i-1}, \forall i \in \{2,\cdots,N\}$

Stability of formation structure of multiple helicopters is achieved by using mesh stability criteria in [143]. In [141], this method is used for stabilizing a multiple surface vessels. String stability notion is a special condition of mesh stability [143].

1.11.3 Leader-to-Formation Stability (LFS)

This stability criterion is employed to check the stability of multiple interconnected nonlinear AUVs in leader-follower approach [148]. This method implements the error amplification technique for stability analysis. Nonlinear gain estimation is established using the errors accomplished with the leader vehicles. It provides a safety bound on formation error; hence one can determine the safety specifications and admissible input to the system.

Formation of a group of AUVs is said to be leader-to-follower stable, if the following conditions are satisfied.

If there exist a class $\mathcal{KL}$ function β and class $\mathcal{K}$ function γ such that for any initial formation error $\tilde{x}(0)$ and for any bounded inputs to the formation leaders w_l, the formation error satisfies $\|\tilde{x}\| \le \beta\left(\|\tilde{x}(0)\|, t\right) + \sum_{l \in L_F} \gamma_l \left(\sup_{[0,t]} \|w_l\| \right)$.

$\beta(r,t)$ and $\gamma_l(r)$ are the transit functions.

The LFS is an extended work of input-to-state stability done in [146]. LFS condition provides more robustness properties than that of the input-to-state stability (IOS) notion.

IOS is an equivalent characterization of Input-to-Formation stability (IFS) [145]. The IOS are explained below [145], [147].

Consider the formation state ξ and the formation performance output z. The formation state is obtained from the original state vectors of the AUV and controllers by a coordinate transformation,

$$\xi = \chi(t, z) \tag{1.26}$$

where $x \triangleq (x_1, x_2, \cdots, x_N)$. The performance output z is a function of the formation state $z = h(t, \xi)$. Then the closed loop formation dynamics are given by

$$\begin{aligned} \dot{\xi} &= f(t,\xi,d) \\ z &= h(t,\xi) \end{aligned} \tag{1.27}$$

where $f(0,\cdots,0)=0$ and $h(t,0)=0$ for all t.

The formation (1.27) is input-to-output stable from d to z with gains ρ and λ defined by class $\mathcal{K}$ functions, if the following conditions are satisfied.

(i) The system (1.27) has well defined solution for all $t \geq 0$, initial conditions $\xi(t_0)$ and inputs $d(\cdot)$.

(ii) All solutions of (1.27) satisfy,

$$\begin{aligned} \|z\|_\infty &\leq \max\left\{\rho\left(\|\xi(t_0)\|\right), \lambda\left(\|d\|_\infty\right)\right\} \\ \limsup_{x\to\infty}\|z(t)\| &\leq \lambda\left(\limsup_{x\to\infty}\|d(t)\|\right) \end{aligned} \tag{1.28}$$

where $\|z\|_\infty \triangleq \sup_{t\geq 0}\|z(t)\|$

These stability criteria help the system to accomplish stable formation structure, provide safety bounded inputs and outputs conditions for the AUV formation system.

1.12 Motivation of the Present Work

- The dynamics of an AUV being highly nonlinear, uncertain in parameters and time varying, thus the development of path following as well as the formation and flocking algorithms are challenging problems. The dynamics of an AUV are highly nonlinear due to the uncertainties in the coupled hydrodynamic damping coefficients. In view of the uncertainties in the dynamics and achieving stability of the AUV during path following, there is a good research opportunity to develop adaptive controllers.
- Having developed the path following control algorithms, these can be extended for formation control algorithms for multiple AUVs put to a group motion and formation.
- Flocking control of multiple mobile robots is reported in literature, which can be extended to development of flocking control algorithms of multiple AUVs. Further, these flocking control algorithms need to incorporate obstacle avoidance logic such

that multiple AUVs in flocking would work in a co-operative manner in an obstacle rich environment.

1.13 Objective of the Thesis

- To develop path following control laws for underactuated AUVs using PD (proportional derivative), Augmented PD and artificial potential functions in an obstacle-free as well as obstacle-rich environment.
- To develop adaptive path following control law for a single AUV as well as adaptive formation control law for a number of AUVs in a group.
- To develop a potential energy function based gravitation compensation PD control law whose control action steers a group of AUVs towards a safety region.
- To develop flocking control laws for a multiple number of AUVs using mathematical as well as fuzzy potential functions both in obstacle-free as well as in the obstacle-rich environments.

1.14 Thesis Organization

Chapter 1 presents the introduction to AUVs, their applications and detailed literature review on path following and cooperative control of multiple AUVs including a taxonomy of cooperative control, different cooperative coordination strategies, cooperative control strategies among autonomous vehicles, network and communication issues and different sub-problems of cooperative control. Subsequently, it provides motivation, objectives of thesis and thesis organisation.

Chapter 2 describes a proportional derivative and augmented proportional derivative controller development for controlling an underactuated AUV along a desired trajectory using potential function approaches. The chapter also provides stability analysis of the developed controller by using the Lyapunov's stability criterion.

Chapter 3 presents the development of an adaptive trajectory tracking based formation control of multiple AUVs together with the stability analysis.

Chapter 4 presents the formation control of multiple AUVs navigating towards a safety region. The travelling area is divided into different regions such as unsafe and safe region. A PD control law is developed which is able to force the AUVs moving towards the safety area.

Chapter 5 describes the development of control law for the flocking of multiple AUVs under obstacles-rich environment using consensus protocols. Flocking algorithm for multiple AUVs is developed using mutual mathematical potential function among AUVs. The controller is developed by considering both with and without communication constraints.

Chapter 6 presents the development of fuzzy potential function based flocking controller for a group of AUVs using consensus algorithms.

Chapter 7 provides overall conclusions with a summary of works presented in previous six chapters. Subsequently suggestions on the future scope of research in extending the work are presented.

Chapter 2

Path Following Control of an AUV in an Obstacle-rich Environment

This chapter presents the development of a simple but powerful path following and obstacle avoidance control law for an underactuated AUV. Potential function based proportional derivative (PFPD) as well as potential function based augmented proportional derivative (PFAPD) control laws are developed to govern the motion of the AUV in an obstacle-rich environment. For obstacle avoidance mathematical potential function is used which formulate the repulsive force between the AUV and the solid obstacles intersecting the desired path. Numerical simulations are carried out to study the efficacy of the proposed controllers and the results are observed. To reduce the values of the overshoots and steady state errors identified due to application of PFPD control, a PFAPD controller is designed which drive the AUV along the desired trajectory. From the simulation results, it is observed that the proposed controllers are able to drive the AUV to track the desired path avoiding the obstacles in an obstacle-rich environment. The results are compared and it is observed that the PFAPD performs better than the PFPD to drive the AUV along the desired trajectory.

2.1 Introduction

Over last two decades, the performance of single AUV and group of multiple AUVs in cooperative play crucial roles in different important cases. The specific applications include [1], [2], [3], security patrols, search and rescue in hazardous environments etc.. Due to the high demand for oil, the price of oil increases day-to-day this causes the economic slump throughout the world. So the importance of energy is reemphasized and searching for new sources of energy is taken into hand. As the importance of energy sources increases, the area of finding that is also increasing and it extends to the deep sea areas. For achieving that, competition among different countries increases. Exploring and developing deep sea areas need different kinds of sensors and devices. Remotely operated vehicles (ROVs) and AUVs are directly fulfilling these requirements [4]. In all these applications a single AUV or a group of multiple AUVs are deployed for fulfilling the sole goal. Most of the applications of the AUVs are more or less integrated with the path planning or path following attribute. Hence, path following control law plays an important role in the operation of AUVs.

The path is planned by considering two points such as start point and a destination point with a velocity field, where the path is to be traversed within minimum time [189], [190]. The optimum path is efficiently generated using path parameterization, cost function techniques and minimum expenditure of energy [190]. Coordinated path following problem of marine craft is proposed in [179], where the path following situation is discussed based on the convergence of geometric errors at the origin of each vehicle. The problem of path planning has been solved based on a fast marching algorithm in [191]. Here, the path tracking algorithm is designed assuming the AUV to manoeuvre on a path of fixed depth. The AUVs are given a task of mine countermeasure (MCM) based on synthetic aperture sonar (SAS) data collected at sea [192]. The path following problem has been solved by getting inspiration from biological agents in [193]. This bio-inspired model in the horizontal plane is capable of generating real time with smooth forward and angular velocities. The path following and trajectory tracking problems are solved by back-stepping approach using a special type of kinematic which is developed by Lyapunov’s direct method and then it is extended to solve the dynamic problem in [194]. The problem of trajectory planning and tracking control is solved by using error dynamics of an AUV in the horizontal plane is presented in [195]. A feedback controller based on LOS in presence of oceanic current disturbances is developed in

[196] to control the AUVs for tracking along the desired trajectories. A region boundary adaptive controller is developed for trajectory tracking of the AUV is presented in [197]. Here the entire boundary is united by the use of multiplicative potential energy functions. In [198], back-stepping controller along with the LOS guidance system is used to control an AUV to track along the desired trajectory in the presence of ocean current. Here the unknown parameters are estimated by using parameter rejection techniques. In [203], a different type of path following problem has been solved for underwater marine vehicles. Here the whole system is divided into a multi-body system. Within that multi-body system the main attribute as well as the relation between individual systems remain unaltered and the whole system move along the desired path.

In case of exploration and exploitation of resources located in deep ocean environment, AUVs play an important role. The AUVs are used in risky and hazardous operations such as bathymetric surveys, oceanographic observations, recovery of lost man-made objects and ocean floor analysis etc.

Hence the path following problem of an AUV is an important case in case of AUVs control. It is assumed that, the dynamic behaviour of an AUV is similar to that of an under actuated surface vessel when moving in a horizontal plane. The orientation of the AUV is also similar to that of the surface vessel. The trajectory tracking requires the design of control law which guides the AUV to track along the desired trajectory. Motion and control of the under actuated AUV are considered in three degrees of freedom. The AUV dynamics are highly nonlinear and coupled together which make the mathematical modelling a hard task. The three dimensional forces and moments of a simple AUV is presented as in Figure 2.1.

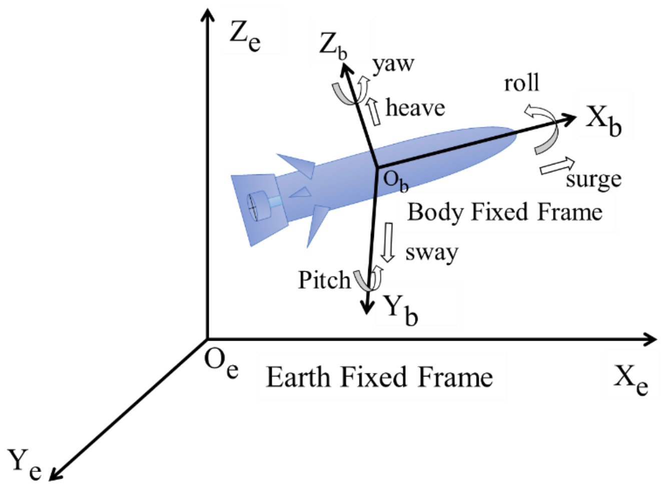


Figure 2.1: Schematic presentation of an AUV with different frame of references

The contribution of the chapter lies in the development of the potential function based PD controller to drive the AUV along the desired path in an obstacle-rich environment. The stability of the proposed control law has been analysed using the Lyapunov's stability criterion.

The organization of the chapter is as follows. Section 2.2 presents the objective of the work. Problem formulation is presented in Section 2.3. AUV kinematics and dynamics are explained in Section 2.4. The potential function based control law is developed and the stability analysis of this controller based on Lyapunov's criterion is presented in Section 2.5. To verify the efficiency of the developed control law, simulation works are presented and the results are discussed in Section 2.6. The conclusions are presented in Section 2.7.

2.2 Objectives of the Chapter

- To develop PFPD and PFAPD path following control laws for an underactuated AUVs based on artificial potential functions both in obstacle-free as well as obstacle-rich environment.
- To analyse the stability of the developed PFPD and PFAPD controllers using Lyapunov's direct stability criterion.

2.3 Problem Formulation

2.3.1 Path Following

Starting from an arbitrary point, after a certain time the AUV has to travel along the desired trajectory. In other words, as time tends to infinite the actual trajectory travelled by the AUV will coincide with the desired one, i.e. the errors between the positions of desired and actual trajectory will be zero, i.e.

$$\lim_{t\to\infty} \left|\eta - \eta_r\right| = 0 \tag{2.1}$$

where t is time, η and η_r are the position vector of the AUV and the position vector of the desired trajectory respectively (Figure 2.2).

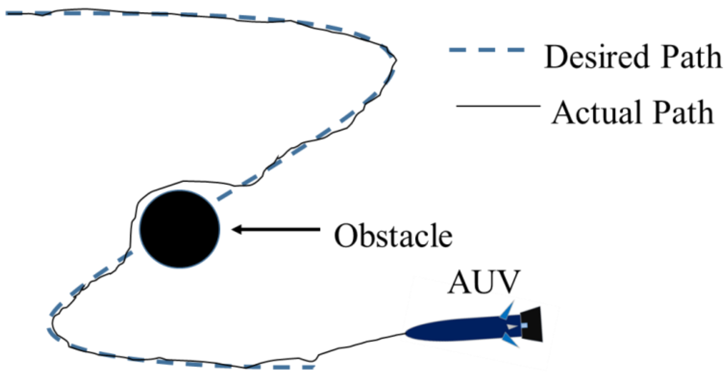


Figure 2.2 Schematic presentation of AUV path following and obstacle avoidance

2.3.2 Obstacle Avoidance

A repulsive force should be established between AUV and the obstacle such that there is no collision happens between AUV and the solid obstacle appears on the desired path. AUV avoid collision with the obstacle and keeps a safety distance from the obstacle. Mathematically, it is denoted as

$$\lim_{t\to\infty}\left\|\eta-\eta_{obs}\right\|\simeq d \tag{2.2}$$

η_{obs} is the position of obstacle, d is the safest distance of AUV form obstacle. The flow chart of the objective and the problem formulation is presented in Figure 2.3.

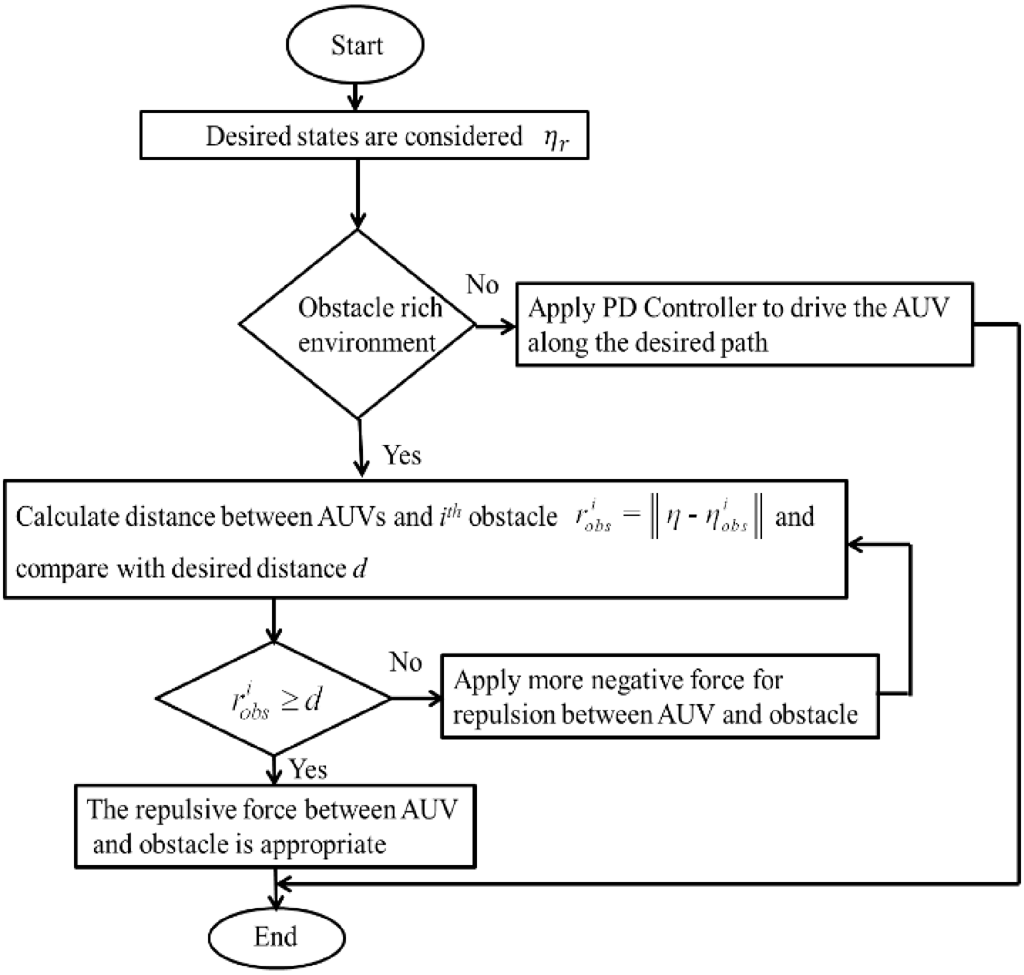


Figure 2.3: Flowchart of path following control and obstacle avoidance of AUV

2.4 AUV Kinematics and Dynamics

2.4.1 AUV Kinematics

In this section, AUV kinematics and dynamics in six degrees of freedom, i.e. the vehicle is moving in space are presented. There are two types of frame of references are taken into account, i.e. body fixed frame of reference $\{B\}$ and another is an earth fixed frame of reference which is known as inertial frame of reference $\{I\}$. The origin of B is coinciding with the centre of mass of the vehicle. The general motion of an AUV in six degrees of freedom (DOF) can be described by the following vectors [34]:

$$\begin{aligned}\eta &= [x, y, z, \varphi, \theta, \psi]^T \\ \nu &= [u, v, w, p, q, r]^T \\ \tau &= [X, Y, Z, K, M, N]^T\end{aligned} \tag{2.3}$$

Here η is the position and the orientation vector with coordinates in the inertial frame. *x*, *y*, *z* are coordinates of position and φ, θ, ψ are orientation along longitudinal, transversal and vertical axes respectively. ν is the velocity vector with coordinates in the body-fixed frame. *u*, *v w* present linear velocities. *p*, *q* and *r* present the angular velocities. *X*, *Y*, *Z* are forces, *K*, *M*, *N* denote moments. $\boldsymbol{\tau}$ denotes the vector of forces and moments acting on the robot in the body-fixed frame.

2.4.2 AUV Dynamics

The nonlinear dynamic and kinematic equations of motion can be expressed as [189]:

$$\begin{aligned}&M\dot{\nu} + C(\nu)\nu + D(\nu)\nu + g(\eta) = \tau \\ &\dot{\eta} = J(\eta)\nu\end{aligned} \tag{2.4}$$

Here M is the inertia matrix including added mass, $C(\nu)$ is the matrix of Coriolis and centripetal terms including added mass. $\mathrm{D}(\nu)$ denotes hydrodynamic damping and lift matrix and $g(\eta)$ is the vector of gravitational forces and moments. $J(\eta)$ is velocity transformation matrix between robot and earth fixed frames.

This transformation matrix can be presented as follows,

$$J(\eta) = \begin{bmatrix} J_1(\eta) & 0_{3\times 3} \\ 0_{3\times 3} & J_2(\eta) \end{bmatrix} \tag{2.5}$$

where

$$J_1(\eta) = \begin{bmatrix} \cos(\psi)\cos(\theta) & -\sin(\psi)\cos(\phi) + \cos(\psi)\sin(\theta)\sin(\phi) & \sin(\psi)\sin(\phi) + \cos(\psi)\cos(\phi)\sin(\theta) \\ \sin(\psi)\cos(\theta) & \cos(\psi)\cos(\phi) + \sin(\phi)\sin(\theta)\cos(\psi) & -\cos(\psi)\sin(\phi) + \sin(\theta)\sin(\psi)\cos(\phi) \\ -\sin(\theta) & \cos(\theta)\sin(\phi) & \cos(\theta)\cos(\phi) \end{bmatrix} \tag{2.6}$$

and

$$J_2(\eta) = \begin{bmatrix} 1 & \sin(\phi)\tan(\theta) & \cos(\phi)\tan(\theta) \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi)/\cos(\theta) & \cos(\phi)/\cos(\theta) \end{bmatrix} \tag{2.7}$$

Assumptions Taken

Due to the presence of complexities in the structure of the AUV, the mathematical derivation for finding the control law is difficult. For the sake of conveniences some assumptions are taken.

- The CM (centre of mass) and CB (centre of buoyancy) of body coincides each other, mass distribution all over the body is homogeneous.
- The hydrodynamic terms of higher order are negligible, pitch, roll motions are neglected.
- Motion is taken as constant depth motion. Heave, pitch, roll motions are neglected.

The vehicle dynamics are analysed in three degrees of freedom in the horizontal plane that consists of surge, sway and yaw motion for a single AUV. In the inertial frame of reference the position of a single AUV can be presented by $\eta = [x, y, \psi]^T$. x, y present the x and y-position in horizontal plane respectively and ψ presents yaw orientation around the z-axis respectively. Similarly, in the body fixed frame of reference the velocity of a single AUV can be presented by $\nu = [u, v, r]^T$. u, v denote the linear velocity along x-axis and linear velocity along y-axis in the horizontal plane and r represents yaw angular velocity around the z-axis respectively. The horizontal kinematic equations of motion in the horizontal plane can be presented as

$$\begin{aligned} \dot{x} &= u\cos(\psi) - v\sin(\psi) \\ \dot{y} &= u\sin(\psi) - v\cos(\psi) \\ \dot{\psi} &= r \end{aligned} \tag{2.8}$$

or

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ r \end{bmatrix} \tag{2.9}$$

where x and y denote the inertial coordinates of CM of the vehicle and u and v present the linear surge and sway speed in the body fixed frame of the vehicle respectively. The orientation of the body is defined by ψ. The dynamics for a neutrally buoyant AUV with three planes of symmetry is expressed by the following differential equations [195]:

$$\begin{aligned}
\dot{u} &= \frac{m_{22}}{m_{11}} vr - \frac{X_u}{m_{11}} u - \frac{X_{u|u|}}{m_{11}} u|u| + \frac{1}{m_{11}} F_u \\
\dot{v} &= -\frac{m_{11}}{m_{22}} ur - \frac{Y_v}{m_{22}} v - \frac{Y_{v|v|}}{m_{22}} v|v| + \frac{1}{m_{22}} F_v \\
\dot{r} &= \frac{m_{11} - m_{22}}{m_{33}} uv - \frac{N_r}{m_{33}} r - \frac{N_{r|r|}}{m_{33}} r|r|
\end{aligned} \tag{2.10}$$

or

$$\dot{v} = \begin{bmatrix} -\frac{X_u}{m_{11}} - \frac{X_{u|u|}}{m_{11}}|u| & \frac{m_{22}}{m_{11}} r & 0 \\ -\frac{m_{11}}{m_{22}} r & -\frac{Y_v}{m_{22}} - \frac{Y_{v|v|}}{m_{22}}|v| & 0 \\ \frac{m_{11} - m_{22}}{m_{33}} v & 0 & -\frac{N_r}{m_{33}} - \frac{N_{r|r|}}{m_{33}}|r| \end{bmatrix} \begin{bmatrix} u \\ v \\ r \end{bmatrix} + \begin{bmatrix} \frac{1}{m_{11}} & 0 \\ 0 & \frac{1}{m_{22}} \\ 0 & 0 \end{bmatrix} F \tag{2.11}$$

where

$$\dot{v} = \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{r} \end{bmatrix}, \quad \tau = \begin{bmatrix} F_u \\ F_v \end{bmatrix} \tag{2.12}$$

The parameters m_{11}, m_{22} are the combined rigid body and added mass terms along x_b-axis and y_b-axis respectively. m_{33} is the combined rigid-body and added moment of inertia along z_b-axis (Figure 2.1). The variable F_u and F_v denote the control force along the surge and sway motion respectively. $X_u X_{u|u|}$ are added masses and X_u, Y_v, N_r are surge linear drag, sway linear drag and yaw linear drag coefficients respectively which are taken as model parameters. Similarly $X_{u|u|}, Y_{v|v|}, N_{r|r|}$ are surge quadratic drag, sway quadratic drag and yaw quadratic drag coefficients respectively which are also taken as model parameters. The control input τ is considered as τ_{sp} for PFPD controller and τ_{ap} for PFAPD controller.

2.5 Development of Control law

There are two controllers, i.e. PFPD and PFAPD control laws developed here for guiding the underactuated AUV along the desired trajectory. Mathematical repulsive potential functions are used for obstacle avoidance.

2.5.1 Development of PFPD Control law

2.5.1.1 Path Following Control

The position error η_e of the AUV during its motion along the desired path is given by

$$\eta_e = \eta_r - \eta \tag{2.13}$$

with $\eta_e = [x_e, y_e, \psi_e]^T$ is the position error vector and $\eta_r = [x_r, y_r, \psi_r]^T$ is the position vector of the desired trajectory. The errors x_e, y_e and ψ_e are defined as follows.

$$\begin{aligned} x_e &= x_r - x \\ y_e &= y_r - y \quad . \\ \psi_e &= \psi_r - \psi \end{aligned} \tag{2.14}$$

Similarly the velocity error ν_e is presented by

$$\nu_e = \nu_r - \nu \tag{2.15}$$

where $\nu_e = [u_e, v_e, r_e]^T$ is the velocity error vector and $\nu_r = [u_r, v_r, r_r]^T$ is the velocity vector of the desired trajectory

with

$$\begin{aligned} u_e &= u_r - u \\ v_e &= v_r - v \quad . \\ r_e &= r_r - r \end{aligned} \tag{2.16}$$

The proportional plus velocity feedback controller, which is identical to the PFPD controller for path following of the AUV may be given as

$$F_{sp} = -K_{sp}\eta_e - K_{sv}\nu_e \tag{2.17}$$

where K_{sp} and K_{sv} are the proportionality and velocity feedback constant vectors respectively.

2.5.1.2 Potential Function based Obstacle Avoidance

To avoid collision between AUV and obstacles there is an RPF should exist within the group. This function possesses maximum value when there is a chance of collision between AUV and obstacles i.e. when $\eta = \eta_{obs}$ and asymptotically converges to zero when $\|\eta - \eta_{obs}\| \geq d$. $\eta_{obs} = [x_{obs}, y_{obs}, \psi_{obs}]^T$ is the position of obstacle, x_{obs} and y_{obs} are the x and y-positions of the obstacle in horizontal plane respectively, ψ_{obs} is the yaw orientation around the z-axis. So the AUV would able to maintain a minimum certain safety distance from the obstacle. The RPF between AUV and i^{th} obstacle is given as $U_{rep}(\eta, \eta_{obs}^i)$, $i = 1, 2, 3, \cdots, n_{obs}$. n_{obs} is the number of obstacles.

The negative gradient of the RPF is the repulsive potential force. In an obstacle-rich environment, there is a repulsive potential exists between AUV and the obstacles. The negative gradient of the potential function is presented by

$$F_{rep} = -\sum_{i=1}^{n_{obs}} \nabla U_{rep}\left(\eta, \eta_{obs}^i\right) \tag{2.18}$$

η_{obs}^i is the position of i^{th} obstacle and $\eta_{obs}^i = [x_{obs}^i, y_{obs}^i, \psi_{obs}^i]^T$, x_{obs}^i and y_{obs}^i are x and y-position of the i^{th} obstacle respectively, ψ_{obs}^i is the yaw orientation around the z-axis of i^{th} obstacle, $U_{rep}\left(\eta, \eta_{obs}^i\right)$ is the repulsive potential between the AUV and i^{th} obstacle.

The RPF can be derived in different ways. In this chapter a special type of mathematical potential function takes into consideration. This potential function between the AUV and the i^{th} obstacle is presented as [200]

$$U_{rep} = \frac{1}{\left(\frac{\left\| r_{obs}^{i} \right\|}{d}\right)^2} + \log\left(\frac{\left\| r_{obs}^{i} \right\|}{d}\right)^2 \tag{2.19}$$

where r_{obs}^{i} is the distance between AUV and i^{th} obstacle.

The negative gradient for x-coordinate may be represented by

$$F_{rep} = -\nabla U_{rep} = -\frac{2\left(x - x_{obs}\right)\left({r_{obs}^{i}}^{2} - d^{2}\right)}{{r_{obs}^{i}}^{4}} \tag{2.20}$$

U_{rep} is positive definite. Hence F_{rep} must be negative definite.

Hence the total control input τ_{sp} is the combination of (2.17) and (2.20) which is given as

$$\tau_{sp} = F_{sp} + F_{rep} \tag{2.21}$$

or

$$\tau_{sp} = -K_{sp}\eta_{e} - K_{sv}\nu_{e} + F_{rep} \tag{2.22}$$

This control law is able to drive the AUV along the desired path avoiding the obstacles. The control structure is presented in Figure 2.4.

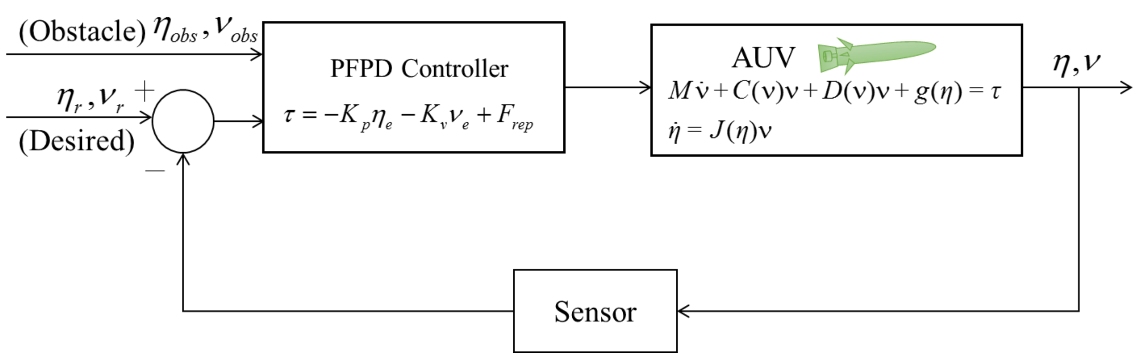


Figure 2.4: Control structure for path following of the AUV

The proposed control law consists of two parts. One is PFPD and another is the potential part. The first part provides the control action to drive the AUV in the desired path. But the second path generates a high level of energy function which is sufficient to avoid the solid obstacles appear on the desired path. So the AUS is able to move safely in the obstacle-rich environment without facing any collision with the obstacles. This can be observed from the simulation study provided in Section 2.6. The stability analysis of the developed control law is presented next.

2.5.1.3 Proof of Stability of the Developed PFPD Control Law

Rearranging (2.10) one can get as

$$\begin{aligned} &m_{11}\dot{u} - m_{22}vr + X_u u + X_{u|u|}u|u| = F_u \\ &m_{22}\dot{v} + m_{11}ur + Y_v v + Y_{v|v|}v|v| = F_v \\ &m_{33}\dot{r} + (m_{22} - m_{11})uv + N_r r + N_{r|r|}r|r| = 0 \end{aligned} \tag{2.23}$$

Eqn. 2.23 can be written in matrix form as

$$\begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & 0 \\ 0 & 0 & m_{33} \end{bmatrix}\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{r} \end{bmatrix} + \begin{bmatrix} (X_u + X_{u|u|}|u|) & -m_{22}r & 0 \\ m_{11}r & Y_v + Y_{v|v|}|v| & 0 \\ 0 & (m_{22} - m_{11})u & N_r + N_{r|r|}|r| \end{bmatrix}\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} F_u \\ F_v \\ 0 \end{bmatrix} \tag{2.24}$$

or

$$M\dot{\nu} + C(\nu)\nu = \tau_{sp} \tag{2.25}$$

where

$$M = \begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & 0 \\ 0 & 0 & m_{33} \end{bmatrix}$$

$$C(\nu) = \begin{bmatrix} (X_u + X_{u|u|}|u|) & -m_{22}r & 0 \\ m_{11}r & Y_v + Y_{v|v|}|v| & 0 \\ 0 & (m_{22} - m_{11})u & N_r + N_{r|r|}|r| \end{bmatrix}$$

and $\tau_{sp} = \begin{bmatrix} F_u \\ F_v \\ 0 \end{bmatrix}$

Substituting the value of τ_{sp} from (2.22) in (2.25) and rearranging it is found as

$$M\dot{\nu} + C(\nu)\nu + K_{sp}\eta_e + K_{sv}\nu_e - F_{rep} = 0 \tag{2.26}$$

The stability of the proposed potential function based simple PFPD control law is proved by using the Lyapunov's stability criterion which is described as below. Choose a Lyapunov candidate function, V_s which satisfies (2.27).

$$\left.\begin{array}{l} V_s(t): \mathbb{R}^n \rightarrow \mathbb{R} \text{ such that} \\ V_s(t) \geq 0 \text{, if and only if } t = 0 \text{ (positive definite)} \\ \dot{V}_s(t) = \dfrac{d}{dt} V_s(t) \leq 0 \text{, if and only if } t = 0 \text{ (negative definite)} \end{array}\right\} \tag{2.27}$$

It is intended to prove that the system is asymptotically stable in the Lyapunov sense if (2.27) is satisfied.

Consider the Lyapunov candidate function V_s as

$$V_s(\eta_e, \nu, t) = \frac{1}{2}\eta_e{}^T K_{sp}\eta_e + \frac{1}{2}\nu^T M \nu \tag{2.28}$$

Computing the time derivative of $V_s(\eta_e, \nu, t)$ from the above equation the stability of the system can be easily proved.

$$\dot{V}_s(\eta_e, \nu, t) = \eta_e{}^T K_{sp}\dot{\eta}_e + \nu^T M \dot{\nu} + \frac{1}{2}\nu^T \dot{M} \nu \tag{2.29}$$

Substituting the value of $M\dot{\nu}$ from (2.26) in (2.29) it is obtained as

$$\dot{V}_s(\eta_e, \nu, t) = \eta_e{}^T K_{sp}\dot{\eta}_e + \nu^T(-C(\nu)\nu - K_{sp}\eta_e - K_{sv}\nu_e + F_{rep}) + \frac{1}{2}\nu^T \dot{M} \nu \tag{2.30}$$

Using $\nu^T = \nu_r{}^T - \nu_e{}^T$ from (2.15), substituting $F_{rep} = -\sum_{i=1}^{N_{obs}} \nabla U_{rep}\left(\eta, \eta_{obs}^i\right)$ from (2.18) in (2.30) and rearranging it can be obtained as

$$\dot{V}_s(\eta_e,\nu,t) = \\ \eta_e{}^T K_{sp}\dot{\eta}_e - \nu^T C(\nu)\nu - \nu_r{}^T K_{sp}\eta_e + \nu_e{}^T K_{sp}\eta_e - \nu^T K_{sv}\nu_e - \nu^T \sum_{i=1}^{N_{obs}} \nabla U_{rep}\left(\eta,\eta_{obs}^i\right) + \frac{1}{2}\nu^T \dot{M}\nu \tag{2.31}$$

Following [201] for fixed trajectory planning $\dot{\eta}_e = -\nu_e$ and for $\eta_e{}^T K_{sp}\dot{\eta}_e = \dot{\eta}_e{}^T K_{sp}\eta_e$, the derivative of Lyapunov function found in (2.31) can be modified as

$$\dot{V}_s(\eta_e,\nu,t) = -\nu^T C(\nu)\nu - \nu_r{}^T K_{sp}\eta_e - \nu^T K_{sv}\nu_e - \nu^T \sum_{i=1}^{N_{obs}} \nabla U_{rep}\left(\eta,\eta_{obs}^i\right) + \frac{1}{2}\nu^T \dot{M}\nu \tag{2.32}$$

Rearranging (2.32) it can be written as

$$\dot{V}_s(\eta_e,\nu,t) = \frac{1}{2}\nu^T\left(\dot{M} - C(\nu)\right)\nu - \nu_r{}^T K_{sp}\eta_e - \nu^T K_{sv}\nu_e - \nu^T \sum_{i=1}^{N_{obs}} \nabla U_{rep}\left(\eta,\eta_{obs}^i\right) \tag{2.33}$$

But $\left(\dot{M} - C(\nu)\right)$ is a skew-symmetric matrix and can be eliminated [202]. So (2.33) became

$$\dot{V}_s(\eta_e,\nu,t) = -\nu_r{}^T K_{sp}\eta_e - \nu^T K_{sv}\nu_e - \nu^T \sum_{i=1}^{N_{obs}} \nabla U_{rep}\left(\eta,\eta_{obs}^i\right) \tag{2.34}$$

All the terms of (2.34) possess negative definite value and hence $\dot{V}(\eta_e,\nu,t) < 0$ which satisfies the Lyapunov stability criterion. Hence the closed-loop velocity feedback system is Lyapunov stable.

2.5.2 Development of PFAPD Control law

2.5.2.1 Path Following Control

The augmented PD controller for path following of the AUV may be given as

$$F_{ap} = M(\eta,\dot{\eta})\ddot{\eta}_r + C(\eta,\dot{\eta})\dot{\eta}_r - K_{ap}\eta_e - K_{av}\nu_e \tag{2.35}$$

where K_{ap} and K_{av} are the proportionality and velocity feedback constant vectors respectively.

As η_r is the desired position and possesses fixed value hence $\dot{\eta}_r$ have the fixed value and will be $\dot{\eta}_r = \nu_r$. Hence the control law (2.35) is presented as

$$F_{ap} = M(\nu)\dot{\nu}_r + C(\nu)\nu_r - K_{ap}\eta_e - K_{av}\nu_e \tag{2.36}$$

2.5.2.2 Potential function based obstacle avoidance

To avoid collision between AUV and obstacles there is an RPF similar to that of used for obstacle avoidance in case of PFPD controller is considered.

Hence the total control input τ_{ap} is the combination of (2.36) and (2.20) which is given as

$$\tau_{ap} = F_{ap} + F_{rep} \tag{2.37}$$

or

$$\tau_{ap} = M(\nu)\dot{\nu}_r + C(\nu)\nu_r - K_{ap}\eta_e - K_{av}\nu_e + F_{rep} \tag{2.38}$$

This is the PFAPD control law to drive the AUV along the desired path avoiding the obstacles. The PFAPD control structure is displayed in Figure 2.5.

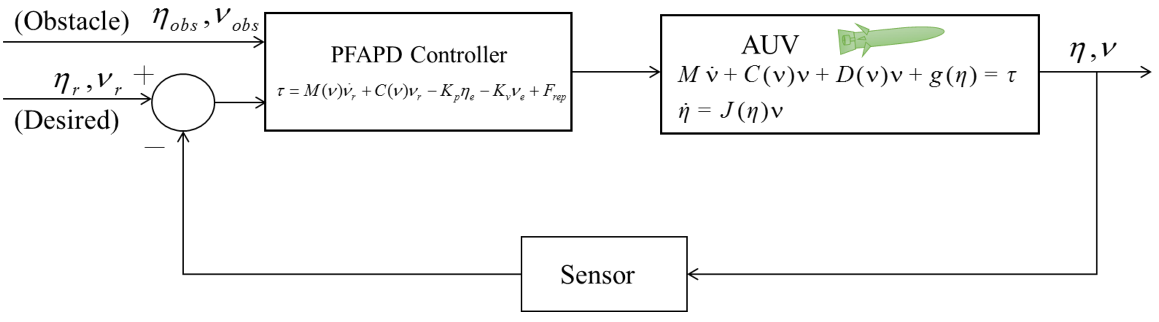


Figure 2.5: Control structure for Augmented PD Controlled path following of the AUV

2.5.2.3 Proof of Stability of the Developed PFAPD Control Law

The dynamic equation of the AUV is considered as

$$M(\nu)\dot{\nu} + C(\nu)\nu = \tau_{ap} \tag{2.39}$$

Substituting the value of τ_{ap} from (2.39) in (2.38), using (2.15) and rearranging it is found as

$$M(\nu)\dot{\nu}_e + C(\nu)\nu_e + K_{ap}\eta_e + K_{av}\nu_e - F_{rep} = 0 \tag{2.40}$$

The stability of the proposed PFPD control law is proved by using the Lyapunov's stability criterion.

Consider the Lyapunov candidate function V_a which satisfies (2.27).

$$\left.\begin{array}{l} V_a(t):\mathbb{R}^n \rightarrow \mathbb{R} \text{ such that} \\ V_a(t) \geq 0 \text{, if and only if } t=0 \text{ (positive definite)} \\ \dot{V}_a(t) = \dfrac{d}{dt}V_a(t) \leq 0 \text{, if and only if } t=0 \text{ (negative definite)} \end{array}\right\} \tag{2.41}$$

It is intended to prove that the system is asymptotically stable if it satisfied the Lyapunov's criterion provided in (2.41).

Consider the Lyapunov candidate function V_a as

$$V_a(\eta_e,\nu_e,t) = \frac{1}{2}\eta_e^T K_{ap}\eta_e + \frac{1}{2}\nu_e^T M(\nu)\nu_e \tag{2.42}$$

Computing the time derivative of $V_a(\eta_e,\nu_e,t)$ from the above equation the stability of the system can be easily proved.

$$\dot{V}_a(\eta_e,\nu_e,t) = \eta_e{}^T K_{ap}\dot{\eta}_e + \nu_e^T M(\nu)\dot{\nu}_e + \frac{1}{2}\nu_e^T \dot{M}(\nu)\nu_e \tag{2.43}$$

Substituting the value of $M(\nu)\dot{\nu}_e$ from (2.40) in (2.43) it is obtained as

$$\dot{V}_a(\eta_e,\nu_e,t) = \eta_e{}^T K_{ap}\dot{\eta}_e + \nu_e^T(-C(\nu)\nu_e - K_{ap}\eta_e - K_{av}\nu_e + F_{rep}) + \frac{1}{2}\nu_e^T \dot{M}(\nu)\nu_e \tag{2.44}$$

As $\dot{\eta}_e = \nu_e$, hence

$$\dot{V}_a(\eta_e,\nu_e,t) = \nu_e^T(-C(\nu)\nu_e - K_{av}\nu_e + F_{rep}) + \frac{1}{2}\nu_e^T \dot{M}\nu_e \tag{2.45}$$

Rearranging (2.45) it is obtained as

$$\dot{V}_a(\eta_e,\nu_e,t) = \frac{1}{2}\nu_e^T\left(\dot{M}(\nu) - C(\nu)\right)\nu_e - \nu_e^T K_{av}\nu_e + F_{rep} \tag{2.46}$$

But $\left(\dot{M}(\nu) - C(\nu)\right)$ is a skew-symmetric matrix and this can be eliminated [202]. So (2.46) became

$$\dot{V}_a(\eta_e,\nu_e,t) = -\nu_e^T K_{av}\nu_e - \nu_e^T \sum_{i=1}^{N_{obs}} \nabla U_{rep}\left(\eta,\eta_{obs}^i\right) \tag{2.47}$$

All the terms of (2.47) possess negative definite value and hence $\dot{V}(\eta_e,\nu_e,t) < 0$ which satisfies the Lyapunov stability criterion. Hence the closed-loop velocity feedback system is Lyapunov stable.

2.6 Results and Discussions

The demonstration of the efficacy of the control law developed in Section 3.4 through simulation study is illustrated here. The desired trajectory is considered as circular path which is described as follows.

$$\begin{aligned} x_r &= 10\sin(0.01t) \\ y_r &= 10\cos(0.01t) \\ \psi_r &= \frac{\pi}{3} \end{aligned} \tag{2.48}$$

The rigid body and hydrodynamic parameters of a special type of AUV i.e. omni directional intelligent navigator (ODIN) are considered and presented in the Table 2.1 [195].

The other parameters are given below.

$$\begin{aligned} m_{11} &= m - X_{\dot{u}} = 215 \text{ kg} \\ m_{22} &= m - Y_{\dot{v}} = 265 \text{ kg} \\ m_{33} &= I_z - N_{\dot{r}} = 80 \text{ kgm}^2 \end{aligned}$$

$$K_{sp} = K_{ap} = [100\ 150\ 0]^T,\ \ K_{sv} = K_{av} = [40\ 50\ 0]^T$$

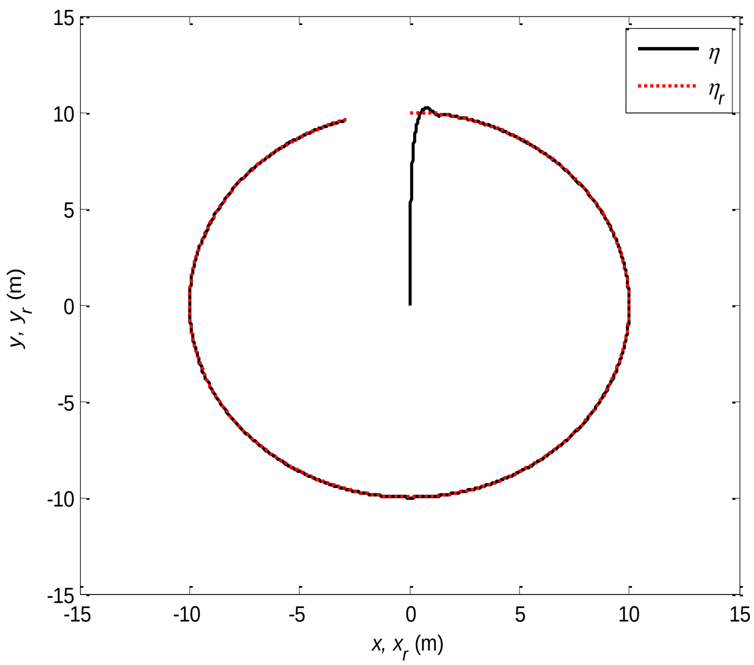
$$\eta^1{}_{obs} = [8,\ 5,\ 0]^T,\ \eta^2{}_{obs} = [-9,\ 5,\ 0]^T$$

Table 2.1: Rigid body and hydrodynamic parameters

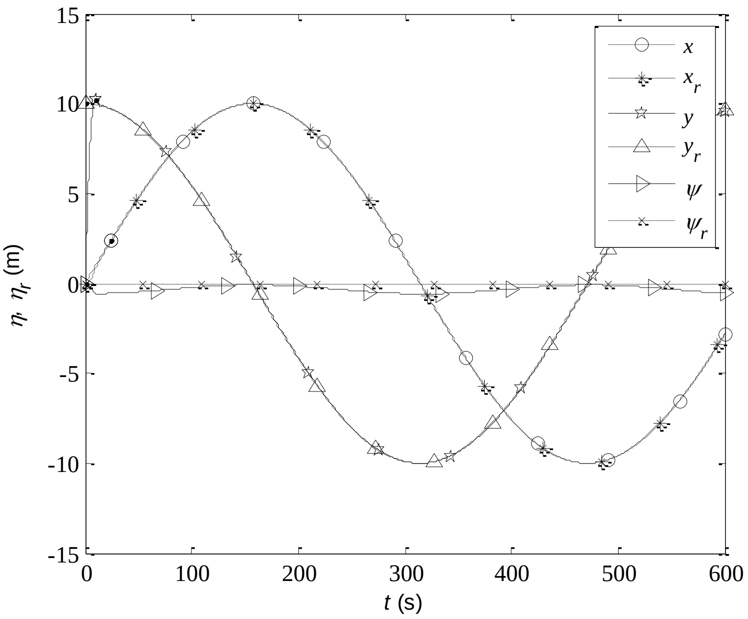
Parameter	Symbol	Value	Unit
Mass	m	185	kg
Rotational mass	I_Z	50	kgm^2
Added mass	$X_{\dot{u}}$	-30	kg
Added mass	$Y_{\dot{v}}$	-80	kg
Added mass	$N_{\dot{r}}$	-30	kgm^2
Surge linear drag	X_u	70	kg/s
Surge quadratic drag	$X_{u\|u\|}$	100	kg/m
Sway linear drag	Y_v	100	kg/s
Sway quadratic drag	$Y_{v\|v\|}$	200	kg/m
Yaw linear drag	N_r	50	kgm^2/s
Yaw quadratic drag	$N_{r\|r\|}$	100	kgm^2

2.6.1 Results due to PD Control Law

The simulation results are presented below.



(a)



(b)

Figure 2.6: (a) Circular trajectory tracking of AUV with desired and actual path (b) matching of actual and desired position

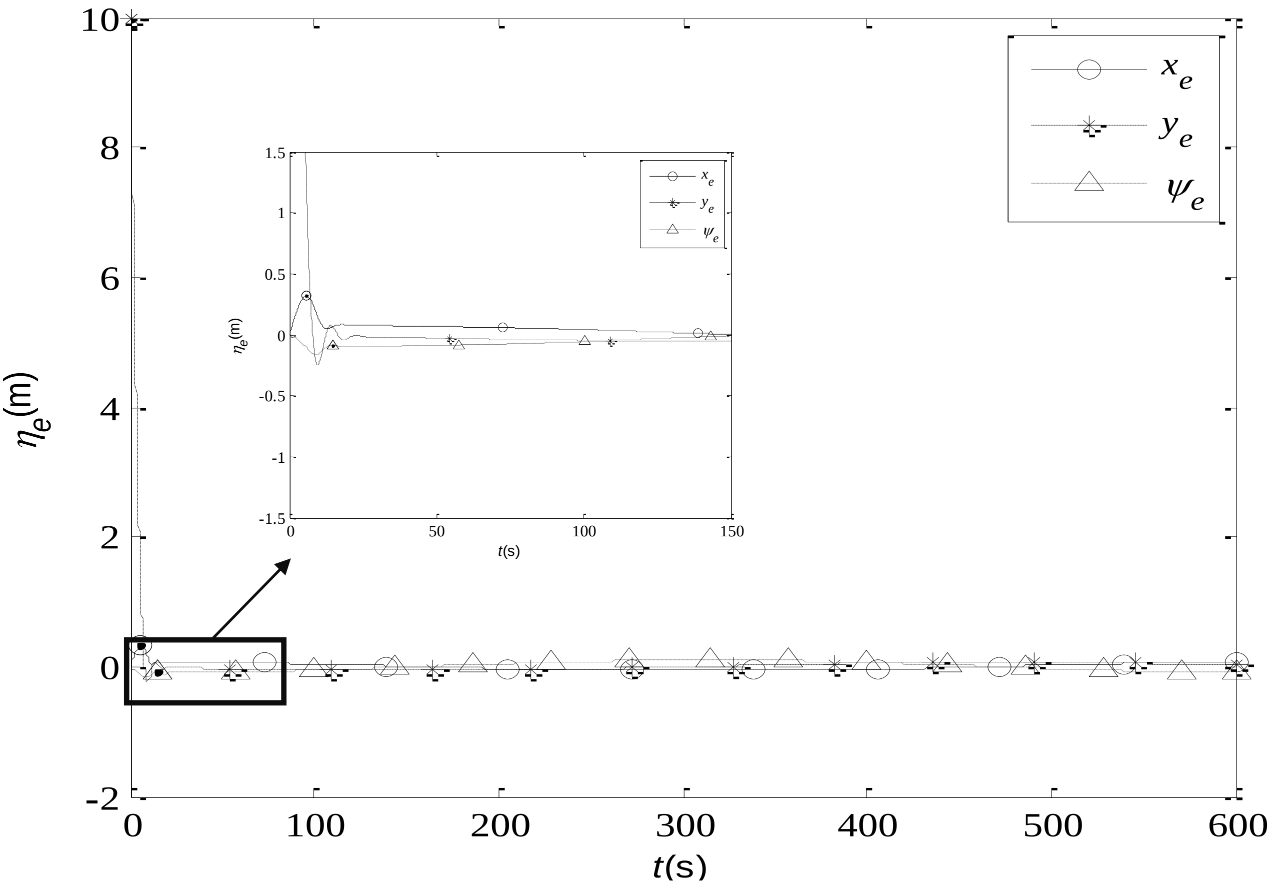


Figure 2.7: Position errors in circular path

Figure 2.6 shows the actual and desired positions when the AUV is intended track a circular path as given in (2.48). From these figures it is observed that, after 10 seconds the AUV traces the desired trajectory as x-position and y-position errors approach zero. This can be verified by observing Figure 2.7. Figure 2.7 depicts the position errors. From these results it is observed that the position errors converge to zero. Thus the proposed controller steers the AUV to track the desired path.

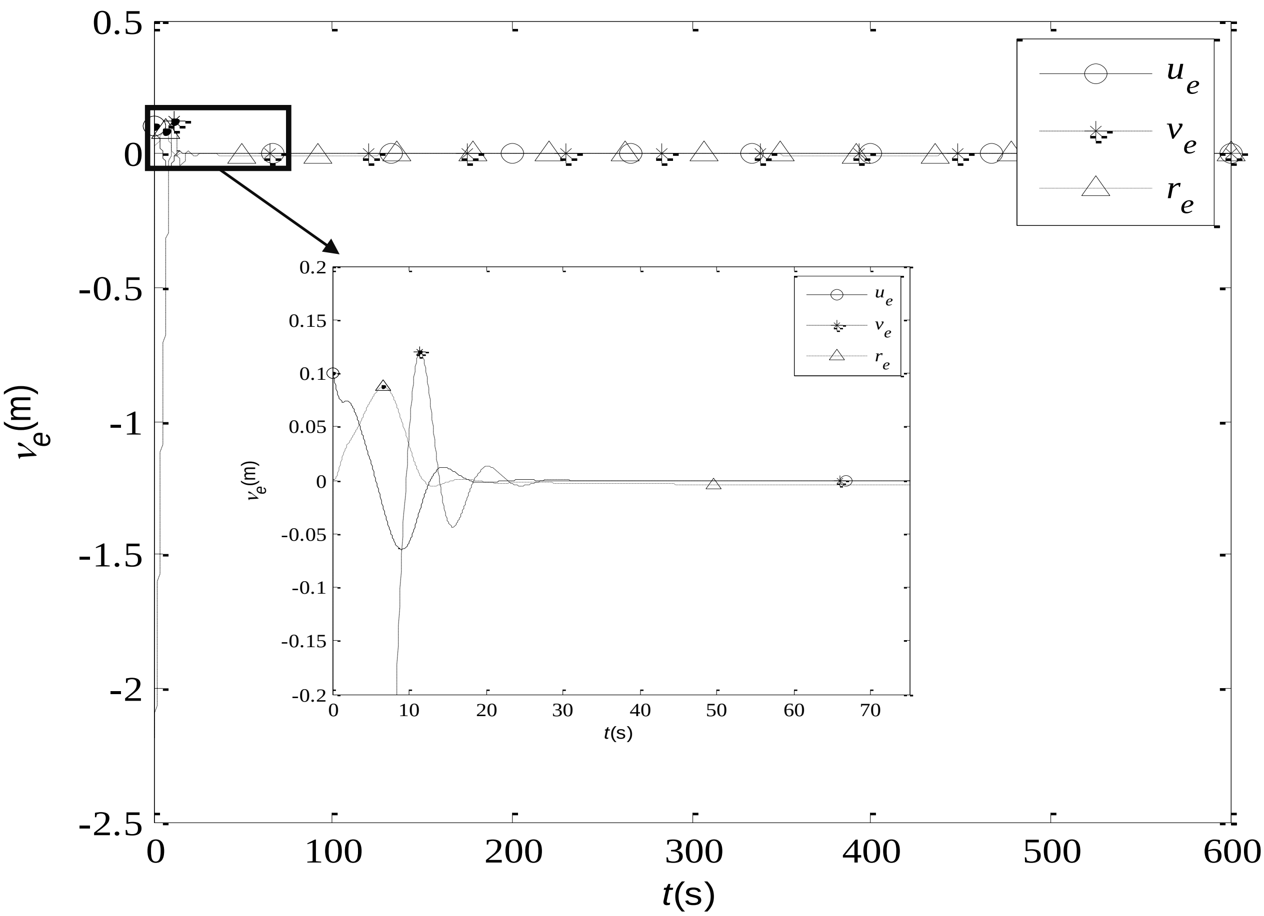
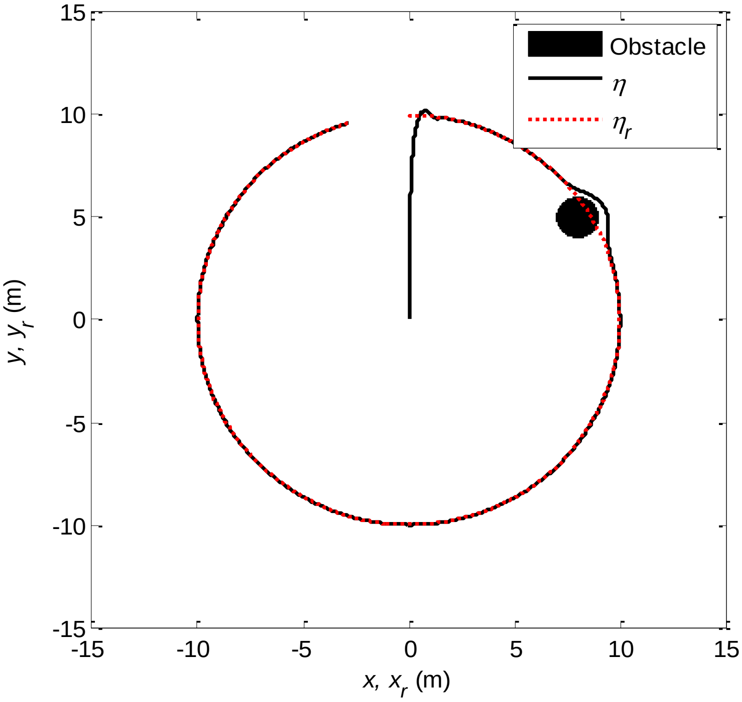


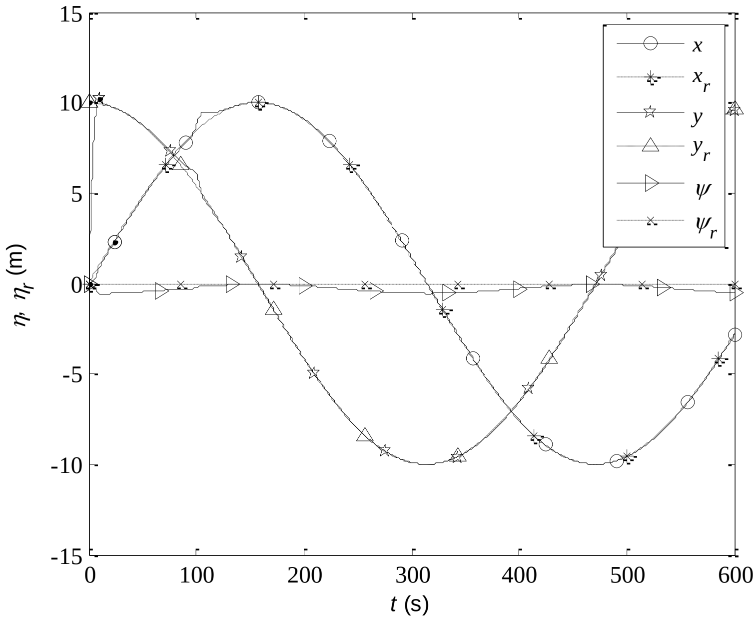
Figure 2.8: Velocity errors in circular path

Figure 2.8 presents the velocity errors. From these results it is observed that the velocity errors converge to zero in about 22 seconds. Hence, after 22 seconds the AUV moves with the desired velocity and processes stable motion.

Comparison of the desired and actual positions as well as orientation of AUV in the earth fixed inertial frame of reference is shown in Figure 2.9. From this figure, it is observed that to avoid the solid obstacle, AUV diverges from the desired path and after passing the obstacle, the AUV again coincide with the desired path.

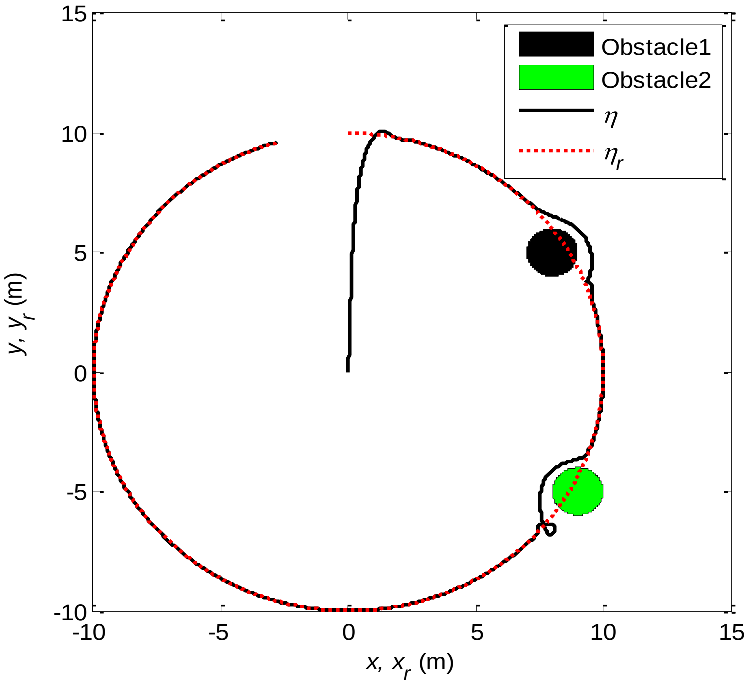


(a)

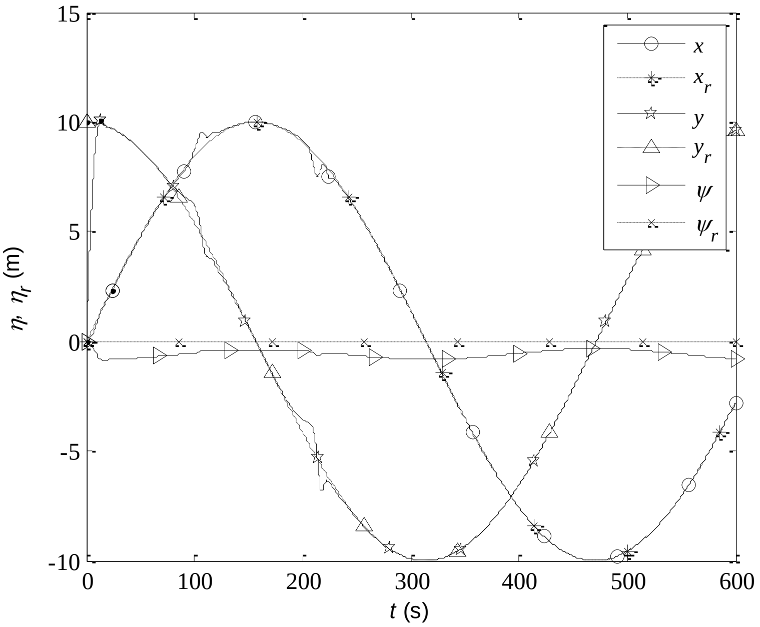


(b)

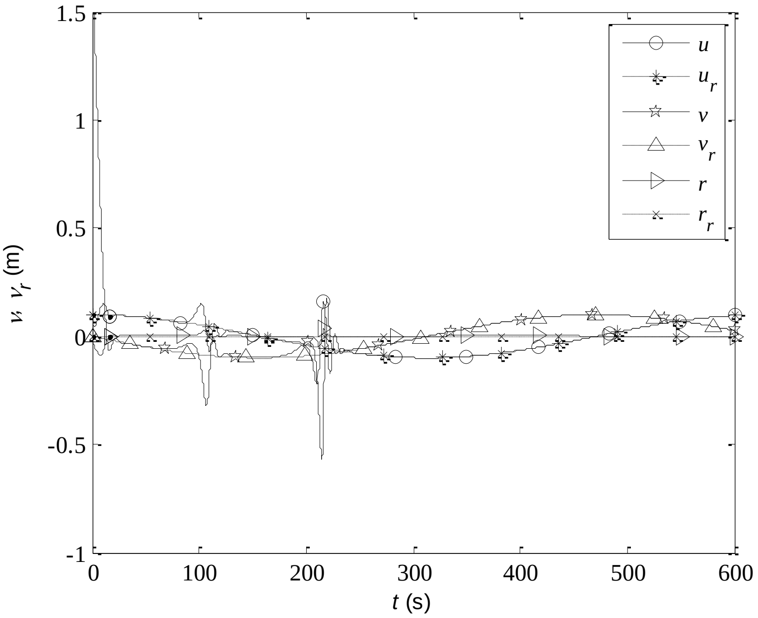
Figure 2.9: (a) Circular trajectory tracking of AUV with desired and actual path with obstacle avoidance (b) Actual position and reference position in a circular path with obstacle avoidance for 600 seconds.



(a)

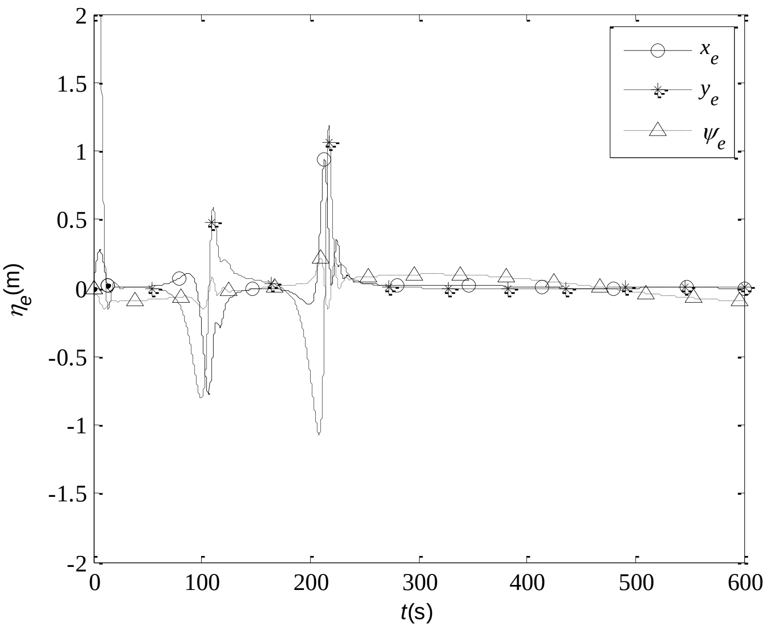


(b)

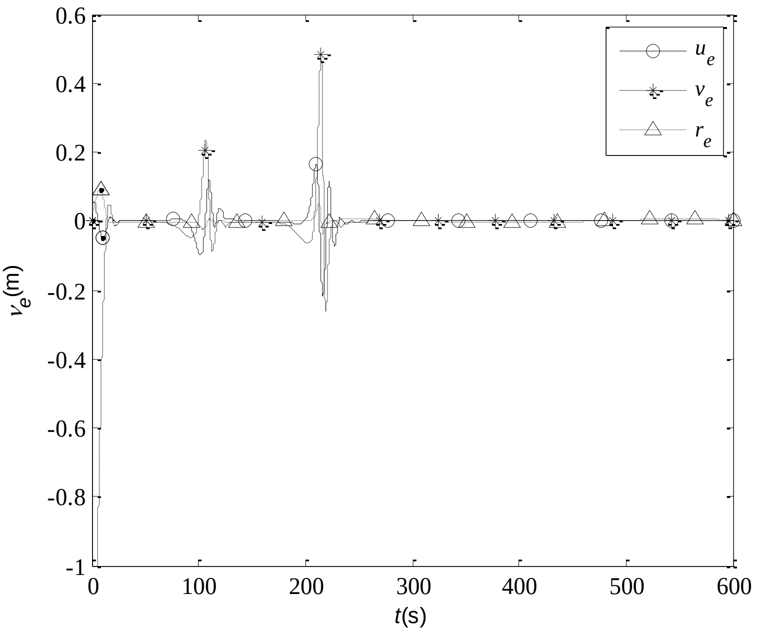


(c)

Figure 2.10: (a) Trajectory tracking of AUV with obstacle avoidance (b) matching of desired and actual positions in obstacle-rich environment (c) velocity matching for 600 seconds



(a)



(b)

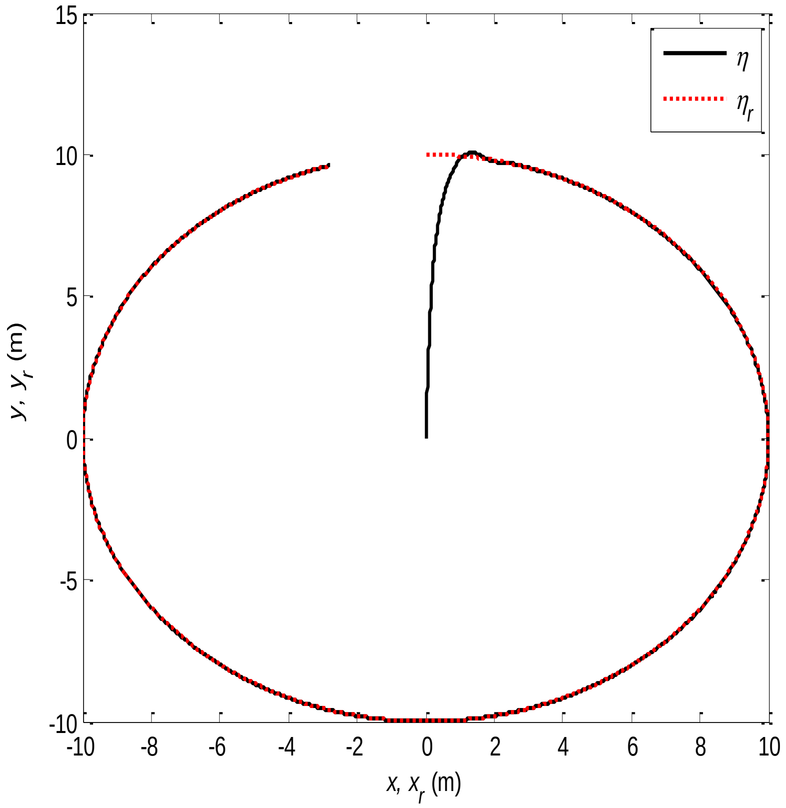
Figure 2.11: (a) Position and (b) velocity errors in circular path for 600 seconds

Figure 2.10(a) shows the motion of the underactuated AUV in a desired circular path in the obstacle-rich region. During its motion, AUV avoids all the obstacles smoothly without colliding with any obstacle. The comparisons of actual and desired positions along the desired path in the obstacle-rich environment are presented in Figure 2.10(b) and Figure 2.10(c). From this figure, it is observed that the actual and desired positions match with each other.

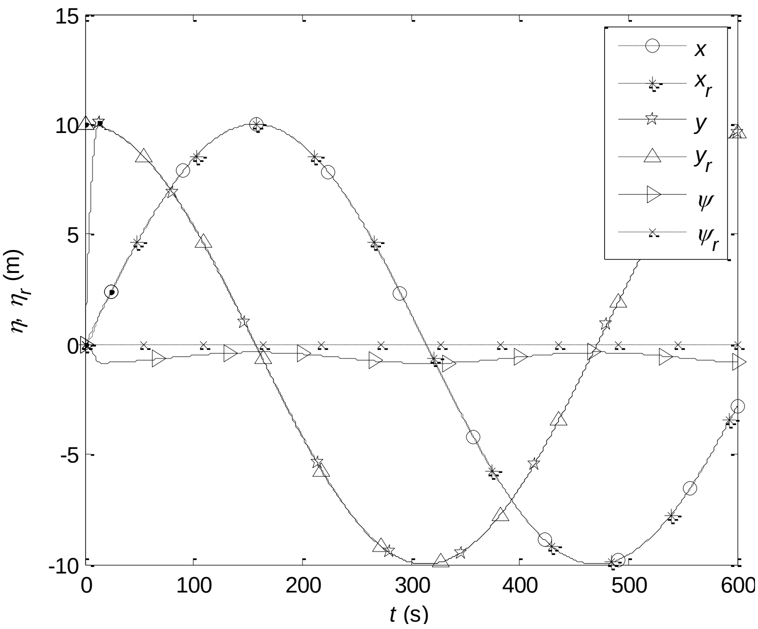
Figure 2.11 displays the position and velocity errors found out during the AUV tracks the desired trajectory. From this figure, it is observed that the position and velocity errors tend to zero in about 23 and 25 seconds respectively. As the AUV diverse much from the desired path during obstacle avoidance, at that particular time the errors appear more obvious.

2.6.2 Results due to PFAPD Control Law

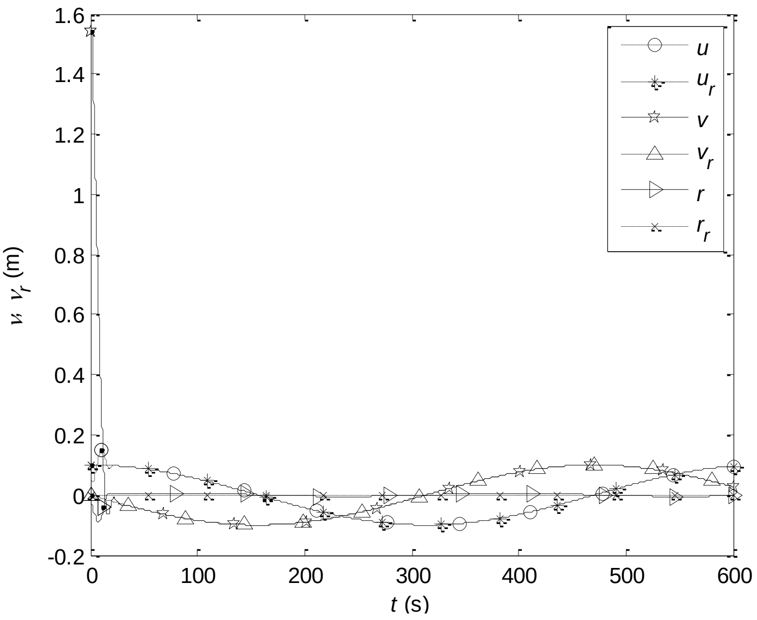
The simulation results due to PFAPD controller on AUV are presented below.



(a)



(b)



(c)

Figure 2.12: (a) Circular trajectory tracking of AUV with desired and actual path (b) matching of actual and desired positions (c) matching of actual and desired velocities

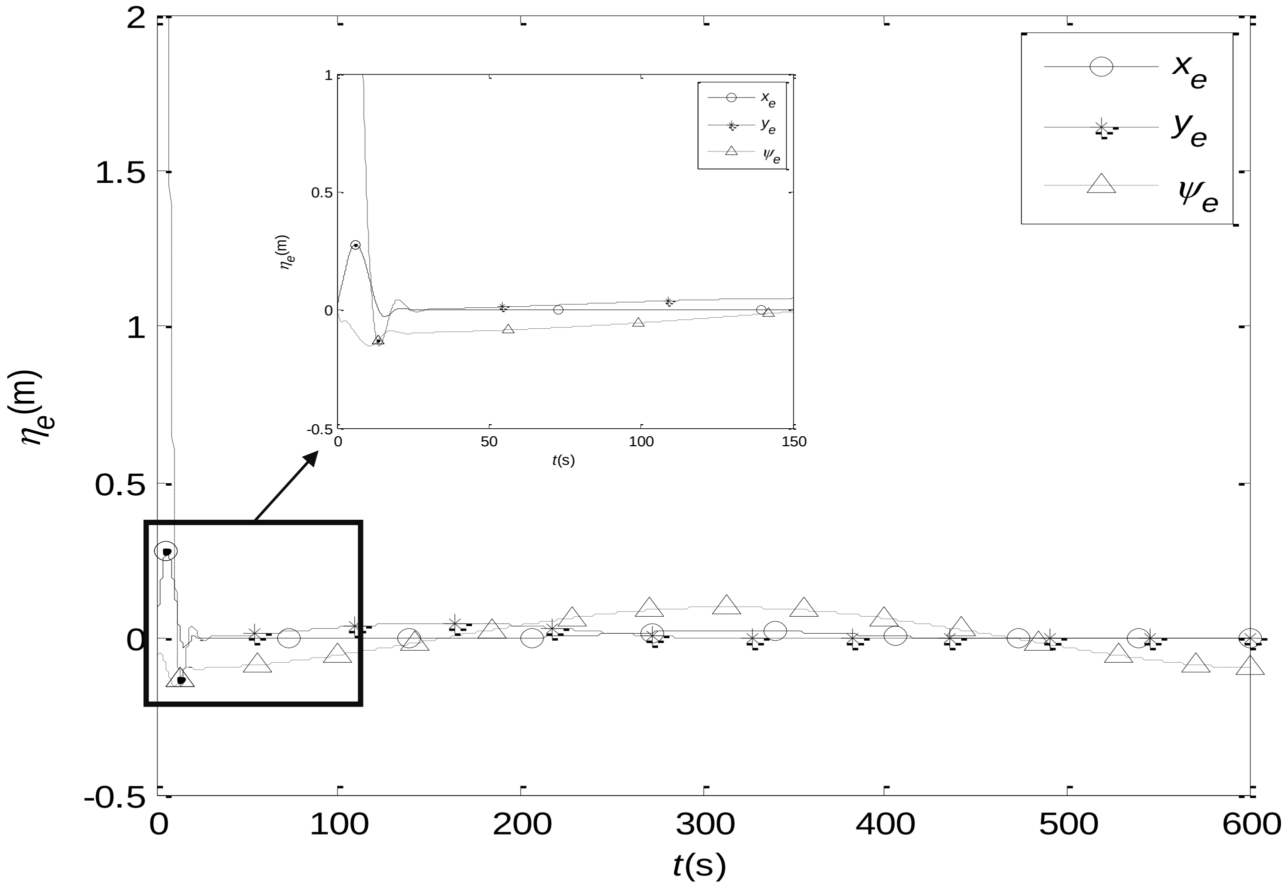


Figure 2.13: Position errors due to application of APD in circular path

Figure 2.12 shows the actual and desired positions when the AUV is intended track a circular path. From these figures it is observed that, after 10 seconds the AUV traces the desired trajectory as x-position and y-position errors approach zero. This can be verified by observing Figure 2.13. Figure 2.13 depicts the position errors. From these results it is observed that the position errors converge to zero. Thus the proposed controller steers the AUV to track the desired path.

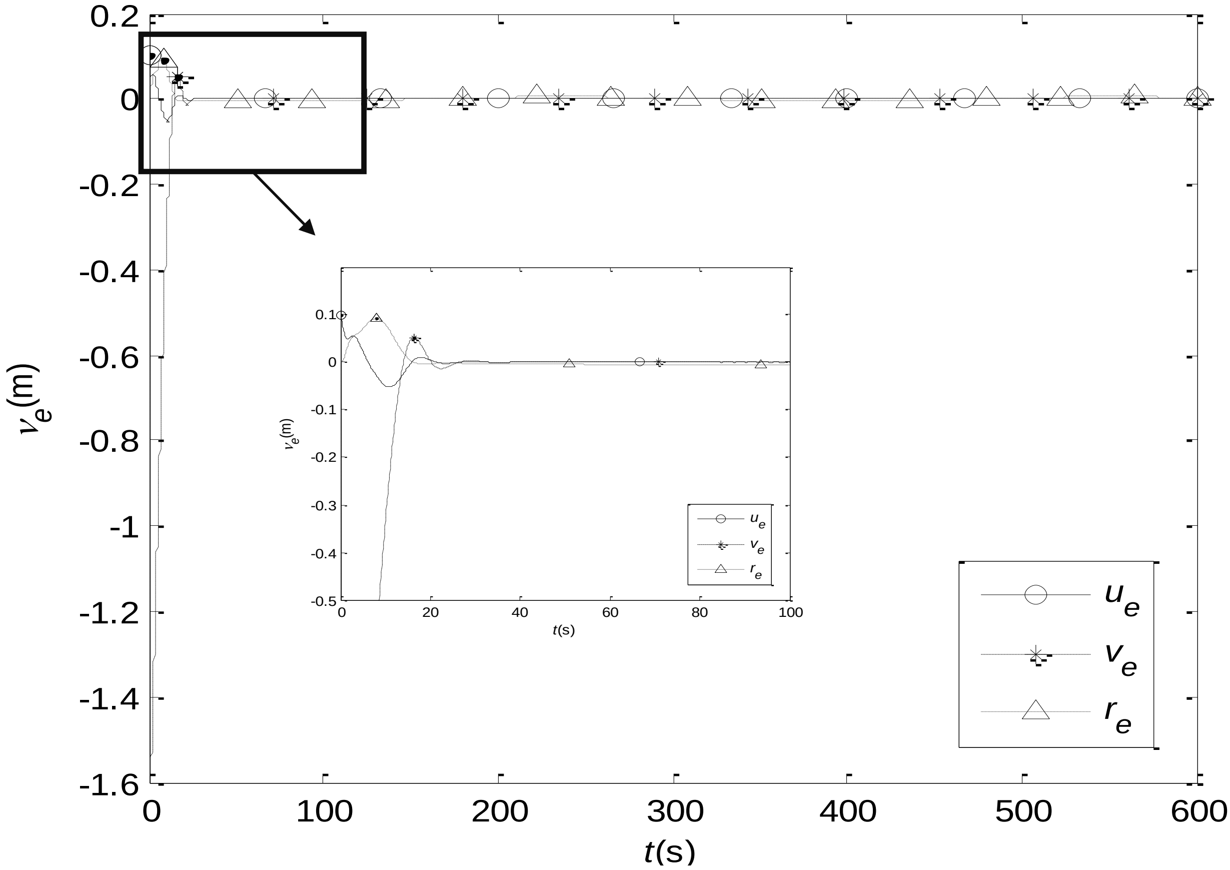
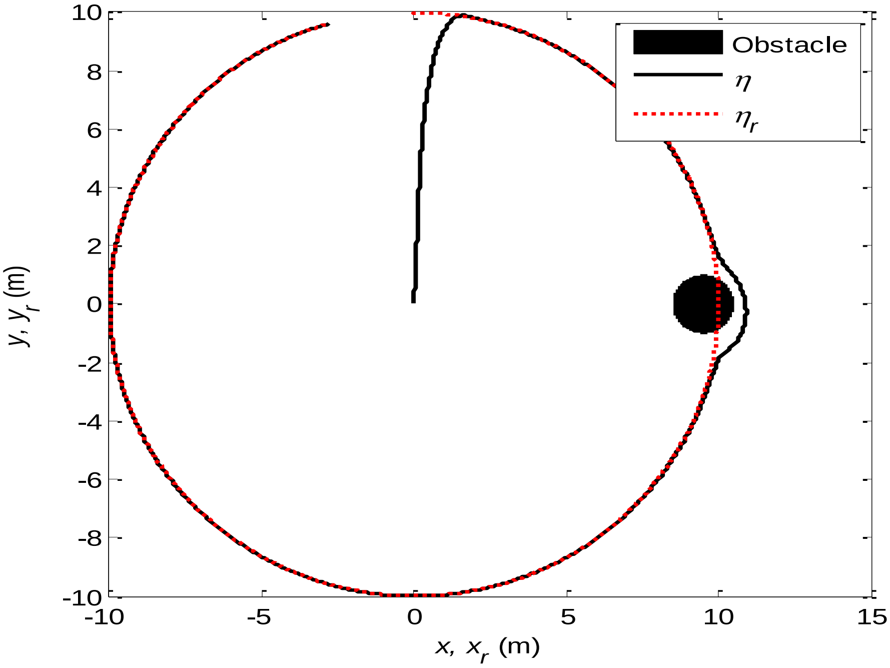
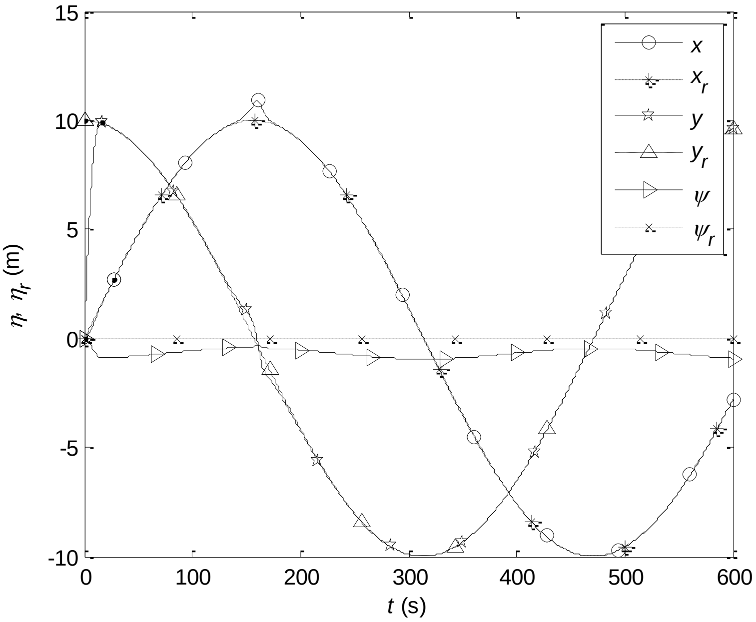


Figure 2.14: Velocity errors due to application of APD in circular path

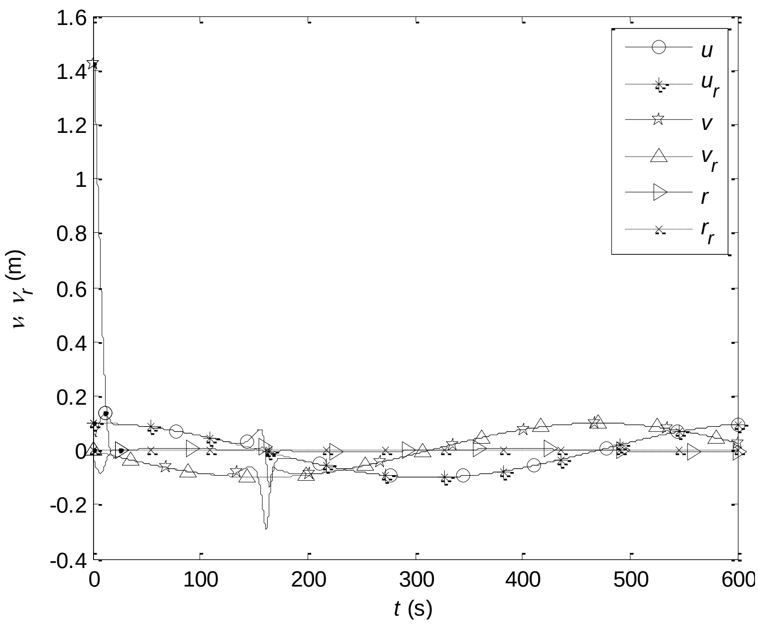
Figure 2.14 presents the velocity errors. From these results it is observed that the velocity errors converge to zero in about 22 seconds. Hence, after 22 seconds the AUV moves with the desired velocity and processes stable motion. The tracking of an underactuated AUV in a desired circular path is shown in Figure 2.15. During its motion, AUV avoids the obstacle smoothly without colliding with it. From this figure, it is observed that to avoid the solid obstacle, AUV diverges from the desired path and after passing the obstacle, the AUV again coincide with the desired path.



(a)

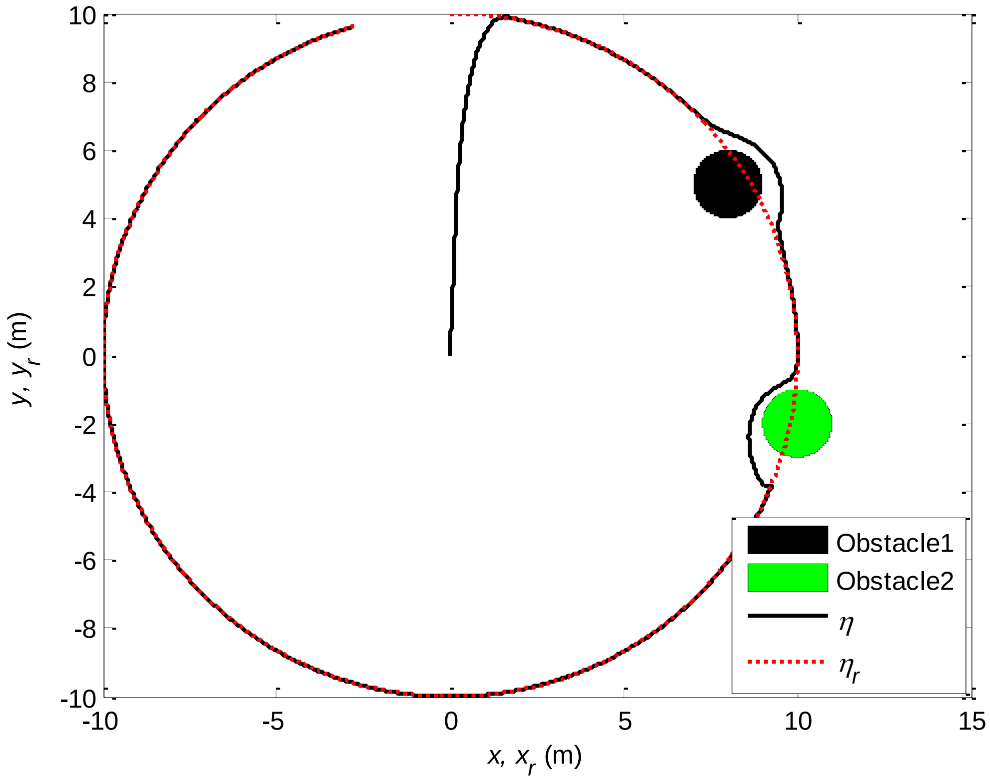


(b)

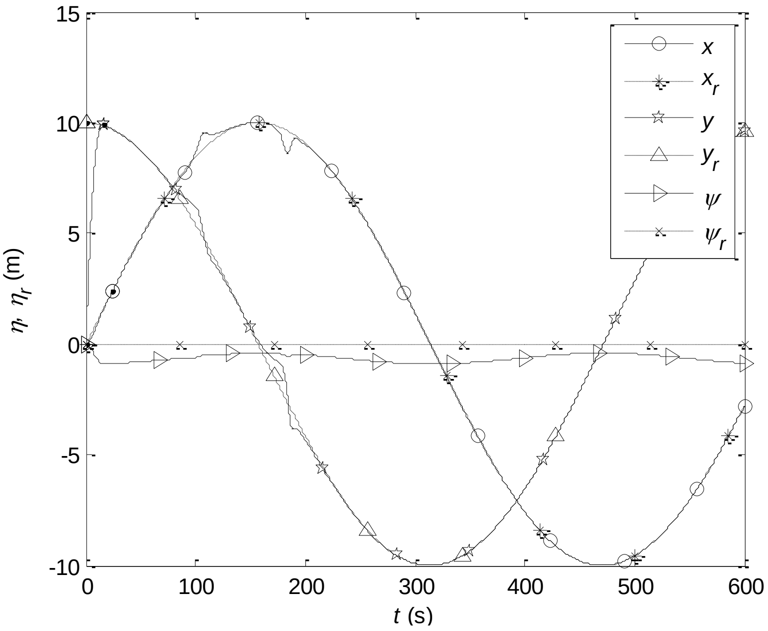


(c)

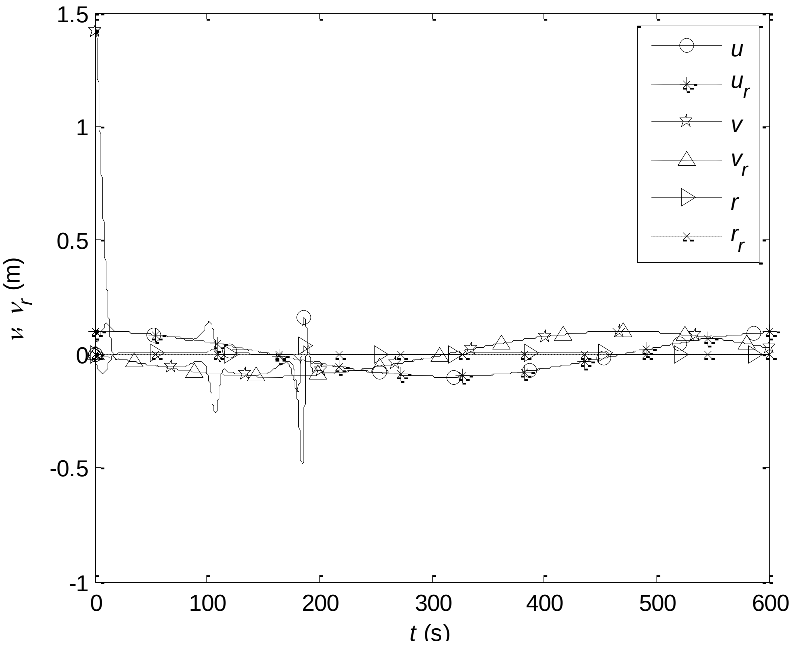
Figure 2.15: (a) Circular trajectory tracking of AUV with desired and actual path with obstacle avoidance (b) actual position and reference position (c) actual velocity and reference velocity in a circular path with obstacle avoidance for 600 seconds



(a)



(b)



(c)

Figure 2.16: (a) Trajectory tracking of AUV with obstacles avoidance (b) matching of desired and actual positions in obstacle-rich environment (c) velocity matching for 600 seconds

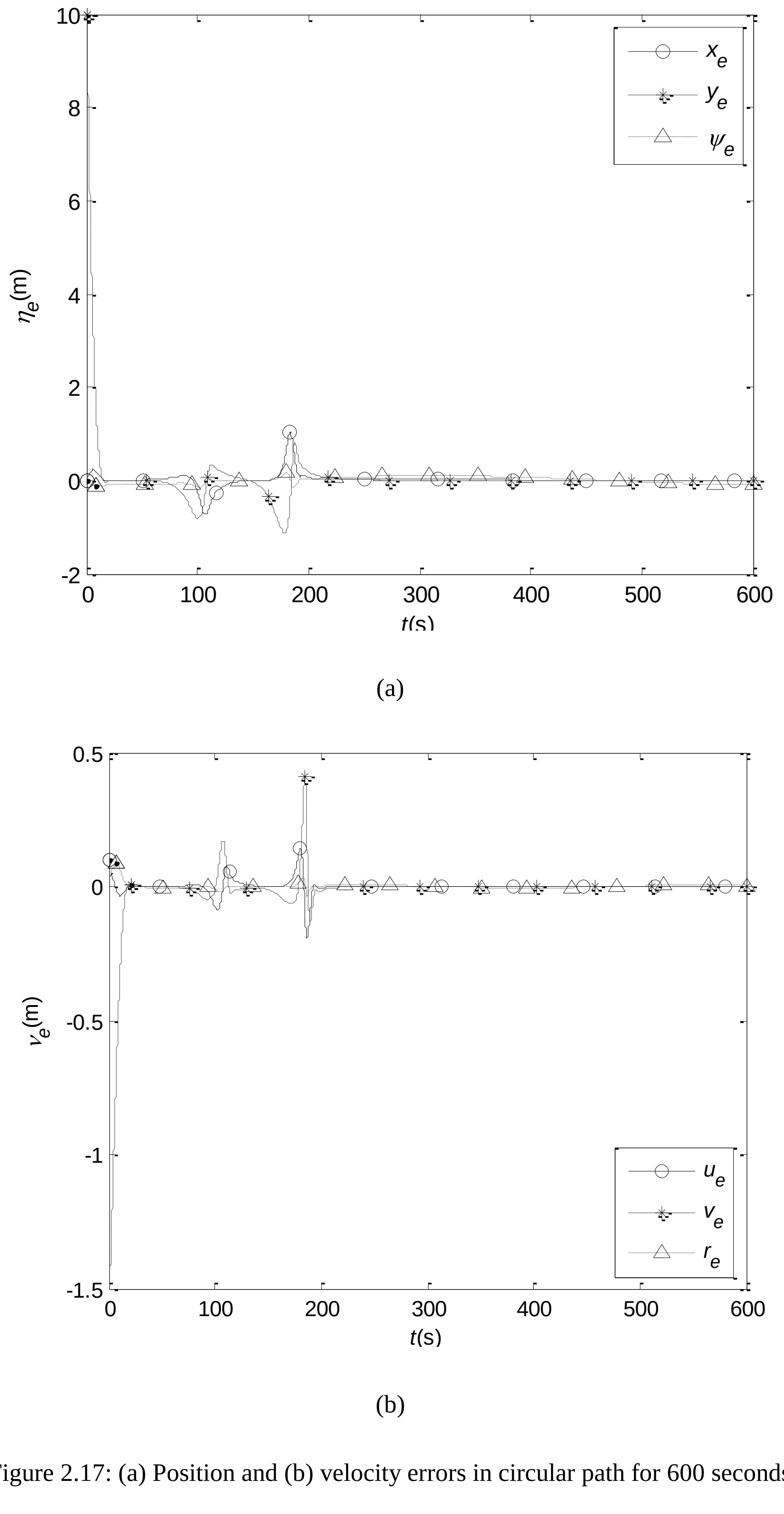


Figure 2.17: (a) Position and (b) velocity errors in circular path for 600 seconds

Figure 2.16(a) shows the motion of the underactuated AUV in a desired circular path in the obstacle-rich region. During its motion, AUV avoids all the obstacles smoothly without colliding with any obstacle. The comparisons of actual and desired positions along the desired path in the obstacle-rich environment is presented in Figure 2.16(b) and Figure 2.16(c). From this figure, it is observed that the actual and desired positions match with each other.

Figure 2.17 displays the position and velocity errors found out during the AUV tracks the desired trajectory. From this figure, it is observed that the position and velocity errors tend to zero in about 23 and 25 seconds respectively. As the AUV diverges from the desired path during obstacle avoidance, at that particular time the errors appear more.

2.6.3 Comparisons

The results obtained due to applications of PFPD and PFAPD are compared. The results are presented in Figure 2.18.

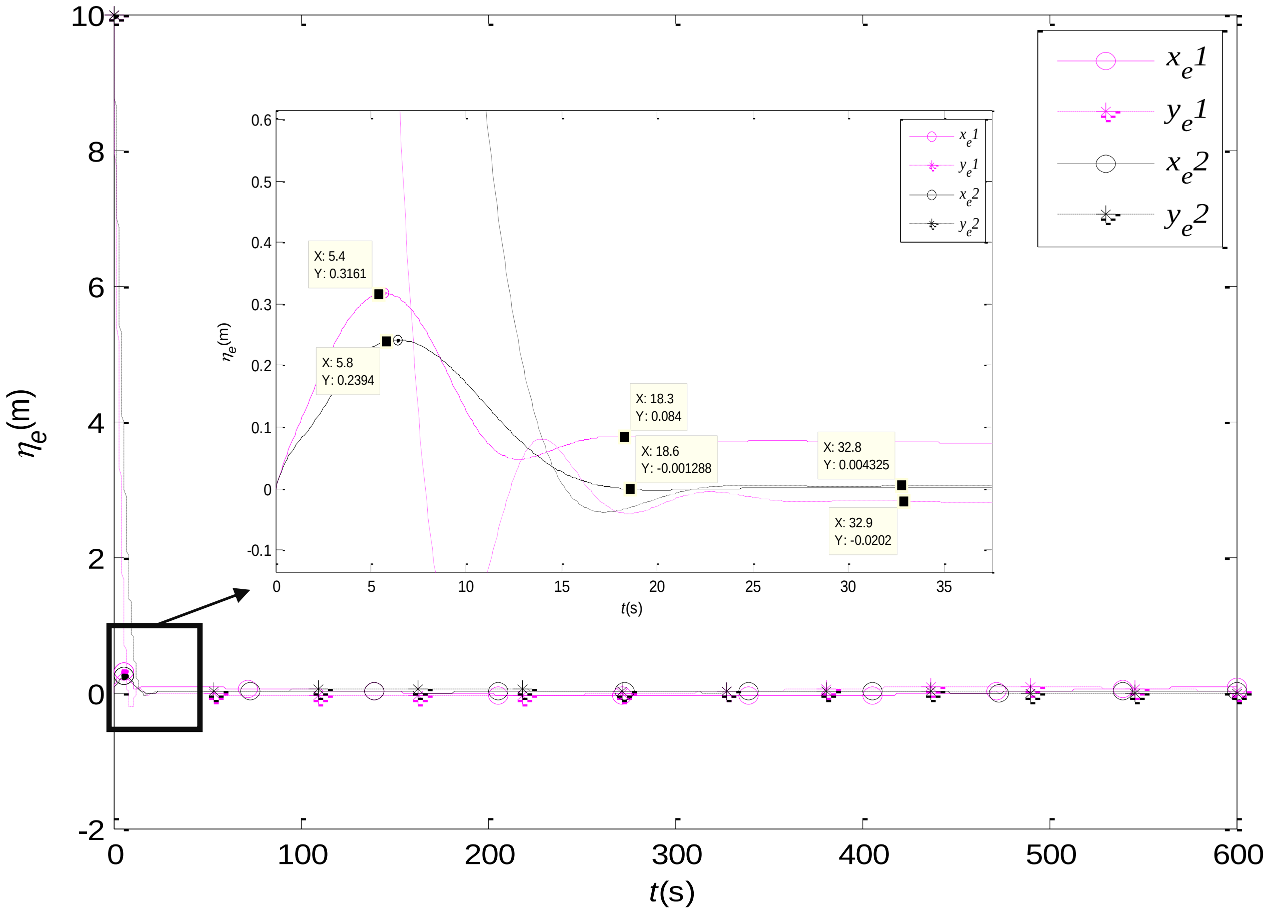


Figure 2.18: Position errors comparison due to allication of PD and APD

From Figure 2.18 it is observed that during tracking along the desired trajectory the errors are asymptotically towards zero. In these figures, x_e1 and y_e1 denote the errors along x and y coordinates of the AUV respectively due to application of PFPD controller. Similarly x_e2 and

y_e2 denote errors due to application of PFAPD. The numerical vales of the first overshoot and the stable errors are provided in Table 2.2.

Table 2.2: Comparison of errors due to application of simple and APD controllers

Control Strategy	Value of Errors (m)			
	First Overshoot		Steady state value	
	x_e	y_e	x_e	y_e
PD	0.03161	10.00	0.084	-0.0202
APD	0.2394	10.00	-0.001281	0.004325

From Table 2.2, it is observed that the error obtained in the first overshoot due to application of PFPD controller is more than due to application of PFAPD. The absolute value of the stable errors observed during PFPD controller are more than that of PFAPD. Hence out of these controllers the PFAPD performs better than a PFPD controller.

2.7 Chapter summary

In this chapter, new PFPD and PFAPD control laws are established for an AUV to track the desired trajectory. The control laws are developed using the RPF among the AUV and the obstacles. The stability of the proposed control laws has been verified us the using of the Lyapunov’s direct stability criterion. Implementing the developed control laws the simulations are carried out, the results are illustrated and explained. Comparing the desired positions with the actual positions it is observed that the errors are asymptotically converging to zero. From the numerical simulation results, the efficacy and accuracy of the developed control laws are verified. The developed control laws are able to force the AUV to track the desired trajectory and capable to avoid the solid obstacles. The results obtained from both these cases are compared and it is observed that, out of these controllers the PFAPD controller exhibits the superior performance.

Chapter 3

Adaptive Trajectory Tracking based Formation Control of AUVs

This chapter presents formation control of multiple AUVs in a distributed manner. The distributed manner in the sense that, each AUV plans its motion based upon the task provided to track along the desired trajectory. An Adaptive formation control law is developed using regressor matrix. The stability of the control law has been verified using Lyapunov stability criterion. This controller considers the hydrodynamic parameter uncertainties of the AUVs. Simulations are carried out considering three AUVs in a group. Obtained results demonstrate the effect of proposed formation control algorithms for a group of AUVs track a circular trajectory.

3.1 Introduction

Formation control of multiple AUVs is an important and popular control problem in recent years owing to their several advantages. Unlike a single vehicle, AUVs in formation, increase the robustness and efficiency of the mission while providing redundancy, reconfiguration ability and structure flexibility [1]. Formation control of AUVs has significant applications in commercial purposes such as in gas and oil industries, in precise surveys or areas where traditional bathymetric surveys are less effective, post-lay pipe surveys and in the military field in the map mines area [204]. As the importance of energy sources increases spontaneously, the area of finding that sources are also increasing and are extend to the deep sea areas. Exploring and developing deep sea areas needed different kinds of sensors and devices. Remotely operated vehicles (ROVs) and AUVs are directly fulfilling these requirements [4]. AUVs play an important role in case of high resolution seabed mapping and commercial survey. In aquaculture, AUVs are used to feed the fishes [7].

Different approaches of formation control are proposed in the literature [205]. Formation control via the behavioural approach [206]; the leader-follower approach [20], [23], [78], [124], [157], [207], [214]; the virtual structure approach [37], [38]; the artificial potential function approach [41], [51], [127], [224]; the graph-theory approach [42]; the geometrical formation control [22], [108], [113], [208]; formation induced by flocking [200], [209], [210], [211], [212]. Other method of formation includes the Lagrangian mechanics has been proposed in [19]; the cyclic pursuit method [45], switching formation [225], reconfiguration formation [213], virtual target tracking formation control [223] etc..

The problems of formation control of multiple AUVs, such as geometric task, dynamic task and synchronization tasks have been solved by using Lyapunov stability based back stepping controller in [180]. Leader-follower algorithm has been developed for multiple AUVs formation using acoustic long base line (LBL) measurements of the position in [181], [215], line-of-sight method in [183], static feedback linearization method in [161], virtual leader concept with L-Si and L-L controllers in [20], virtual leader concept with communication delay in [12], [216]. Formation control of AUVs and obstacle avoidance, which is based on the potential function approach, has been proposed in [21], formation control of multiple AUVs with sea current disturbance are explained in [186]. A formation control law of multiple AUVs using fixed interaction topology criteria has been developed in [178], where an adaptive sliding variable structure control law has been designed for controlling this formation. A nonlinear formation keeping and mooring control of multiple AUVs in chains form has been proposed in [23], where the leader follower method of formation control employing integrator back stepping has been used. Formation control of multiple AUVs considering different communication constraints and the time delay are explained in the continuous domain in [65], [153], [158], [156], [217], in the discrete domain in [222].

The above investigations have not considered the uncertainties due to hydrodynamic damping effects. But it is important to consider these uncertainties to achieve good formation performance. Hence, the hydrodynamic effects are considered and a new adaptive control law is developed for the formation control of multiple AUVs in this chapter using Lyapunov stability criterion. The formation control is a behaviour-based formation control because individual AUV is assigned different tasks of trajectory tracking and as a whole the group completes a single mission [156], [206].

The organization of the chapter is as follows. Section 3.3 describes the problem formulation. AUV kinematics and dynamics were reviewed in brief and an adaptive control law for the formation of multiple AUVs is developed in Section 3.4. In this section the stability of the developed controller has been proved using Lyapunov criterion. To verify the efficacy of the control law for the formation AUVs, simulation results are presented in Section 3.5. Chapter summary is presented in Section 3.6.

3.2 Objectives of the Chapter

- To develop adaptive formation control law for a group of AUVs based on regressor matrix.
- To analyse the stability of the developed adaptive trajectory tracking controller using Lyapunov's direct stability criterion.

3.3 Problem Formulation

Consider Figure 3.1 depicting three AUVs intended to move along a circular path. Starting from any arbitrary points in space, the AUVs need to track the desired circular trajectories maintaining formation. i.e. the errors between the positions of the desired trajectories and actual positions of AUVs should tend to zero at $t \to \infty$.

i.e

$$\lim_{t \to \infty} |\eta - \eta_r| = 0 \tag{3.1}$$

where

η = position and orientation vector of AUV

η_r = desired trajectory and orientation position vector

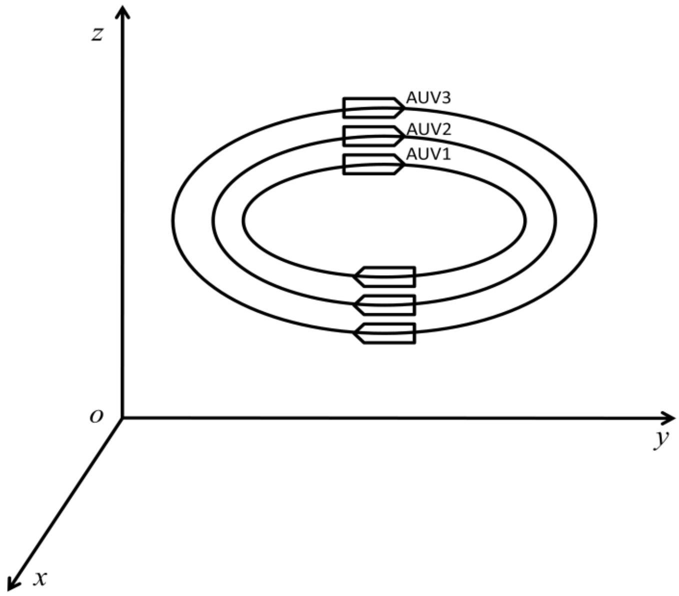


Figure 3.1: Schematic representation of formation AUVs moving along circular trajectories

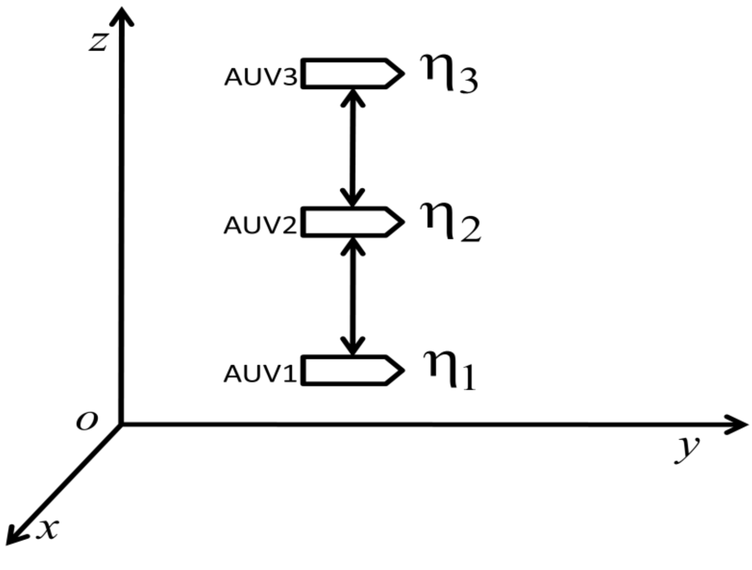


Figure 3.2: Schematic presentation of positions of three AUVs maintaining same distance between them during formation

During the formation, it is necessary that the distance between AUV1 and AUV2 should be equal to that of the distance between AUV2 and AUV3 i.e.

$$\lim_{t\to\infty} |\eta_1 - \eta_2| = |\eta_2 - \eta_3| \tag{3.2}$$

where η_1, η_2, η_3 are position vectors of AUV1, AUV2 and AUV3 respectively (Figure 3.2).

3.4 Development of Adaptive Formation Control law

To develop an adaptive control law for achieving the successful formation, a brief review on AUV kinematics and dynamics are presented in this section first.

3.4.1 Kinematics and Dynamics of an AUV

The kinematics and dynamics of an AUV in six degrees of freedom considering motion in three dimensional space are briefly reviewed here. Two reference frames, namely body fixed frame of reference $\{B\}$ and earth fixed frame of reference or inertial frame of reference $\{I\}$ are considered. The origin of B coincides with the centre of mass of the AUV.

Let there are n number of AUVs are considered in the formation group. The motion of ith AUV $(i = 1, 2, 3, ..., n)$ in six DOF can be described by the following vectors [34].

$$\begin{aligned}
\eta_i &= [x_i, y_i, z_i, \phi_i, \theta_i, \psi_i]^T \\
v_i &= [u_i, v_i, w_i, p_i, q_i, r_i]^T \\
\tau_i &= [X_i, Y_i, Z_i, K_i, M_i, N_i]^T
\end{aligned} \tag{3.3}$$

where η_i is the position and orientation vector of i^{th} AUV in the inertial frame. x_i, y_i, z_i are the coordinates of position of ith AUV and ϕ_i, θ_i, ψ_i are orientation of i^{th} AUV along longitudinal, transversal and vertical axes respectively. v_i is the velocity vector with coordinates in the body-fixed frame. u_i, v_i, w_i denote linear velocities p_i, q_i, r_i are angular velocities. X_i, Y_i, Z_i are forces, K_i, M_i, N_i denote moments. τ_i is the vector of forces and moments acting on the i^{th} AUV in the body-fixed frame.

The kinematic equation of motion is expressed as:

$$\dot{\eta}_i = J_i(\eta_i) v_i \tag{3.4}$$

The dynamic equation of motion of the AUV is presented as

$$M_i(\eta_i)\ddot{\eta}_i + C_i(\eta_i,\dot{\eta}_i)\dot{\eta}_i + D_i(\eta_i,\dot{\eta}_i)\dot{\eta}_i + g_i(\eta_i) = \tau_i \tag{3.5}$$

where $M_i(\eta_i)$ is the inertia matrix including added mass, $C_i(\eta_i,\dot{\eta}_i)$ is the matrix of Coriolis and centripetal terms including added mass. $D_i(\eta_i,\dot{\eta}_i)$ denotes hydrodynamic damping and lift matrix and $g_i(\eta_i)$ is the vector of gravitational forces and moments. $J_i(\eta_i)$ is velocity transformation matrix between the AUV and earth fixed frames.

This transformation matrix between body fixed reference frame and earth fixed reference frame is given by

$$J_i(\eta_i) = \begin{bmatrix} J_{i1}(\eta_i) & 0_{3\times3} \\ 0_{3\times3} & J_{i2}(\eta) \end{bmatrix} \tag{3.6}$$

where

$$J_{i1}(\eta_i) = \begin{bmatrix} \cos(\psi_i)\cos(\theta_i) & -\sin(\psi_i)\cos(\phi_i) + \cos(\psi_i)\sin(\theta_i)\sin(\phi_i) & \sin(\psi_i)\sin(\phi_i) + \cos(\psi_i)\cos(\phi_i)\sin(\theta_i) \\ \sin(\psi_i)\cos(\theta_i) & \cos(\psi_i)\cos(\phi_i) + \sin(\phi_i)\sin(\theta_i)\cos(\psi_i) & -\cos(\psi_i)\sin(\phi_i) + \sin(\theta_i)\sin(\psi_i)\cos(\phi_i) \\ -\sin(\theta_i) & \cos(\theta_i)\sin(\phi_i) & \cos(\theta_i)\cos(\phi_i) \end{bmatrix} \tag{3.7}$$

and

$$J_{i2}(\eta_i) = \begin{bmatrix} 1 & \sin(\phi_i)\tan(\theta_i) & \cos(\phi_i)\tan(\theta_i) \\ 0 & \cos(\phi_i) & -\sin(\phi_i) \\ 0 & \sin(\phi_i)/\cos(\theta_i) & \cos(\phi_i)/\cos(\theta_i) \end{bmatrix} \tag{3.8}$$

Assumptions-1

Due to the presence of asymmetrical complexities in body structure of the AUV, deriving control law is difficult. For the sake of the conveniences the following assumptions are taken. These are as follows.

- CM (centre of mass) and CB (centre of buoyancy) coincides each other. Mass distribution all over the body is homogeneous.
- The hydrodynamic terms of higher order as well as pitch and roll motions are negligible.

Assumptions-2

The behavior based formation control integrates several goals oriented tasks separately [206], [218]. The following assumptions are adapted to solve the problems formulated in this chapter.

- Each AUV can estimate or measure its own position.
- Each AUV is autonomous and it does not rely upon the ant central computationally unit.
- Individual AUV does not know the total number patrolling units present in the group.
- Failure of any AUV does not have any effect to the formation pattern of the AUVs.
- Each AUV knows the geometric description of its desired path locally and separately.
- Any kind of explicit communication in the group of AUVs is forbidden.

3.4.2 Formation Control Law

Parametric uncertainties arise in the AUV dynamics owing to hydrodynamic damping effects. These uncertainties need to be compensated. The adaptive control law is therefore developed to achieve good cooperative behaviour of a system of AUVs in the presence of the above uncertainties by estimating these parameters in real time. An adaptation mechanism is developed to adapt the parameters in the proposed formation control law. An adaptive controller is designed so that it will force the AUVs to track the desired trajectories. For this, define a regressor matrix $Y_i = Y_i(\eta_i, \dot{\eta}_i, \dot{\eta}_{r_i}, \ddot{\eta}_{r_i})$ such that

$$M_i \ddot{\eta}_{ir} + C_i(\eta_i, \dot{\eta}_i)\dot{\eta}_{ir} + D_i(\eta_i, \dot{\eta}_i)\dot{\eta}_{ir} + g_i(\eta_i) = Y_i(\eta_i, \dot{\eta}_i, \dot{\eta}_{ir}, \ddot{\eta}_{_i r})\alpha \tag{3.9}$$

where $\eta_{ir} = [x_{ir}, y_{ir}, z_{ir}, \varphi_{ir}, \theta_{ir}, \psi_{ir}]^T$ denotes the position and orientation vector of the reference trajectory with coordinates in the inertial frame. $v_{ir} = [u_{ir}, v_{ir}, w_{ir}, p_{ir}, q_{ir}, r_{ir}]^T$ is the velocity and angular velocity vector of the reference trajectory.

Let the control law has the following structure

$$\tau_i = Y_i \hat{\alpha} - K_D s_i \tag{3.10}$$

where $Y_i \hat{\alpha} =$ Feed forward term, $K_D s_i =$ a simple PD term. K_D is a positive definite gain matrix and s_i = error vector.

$$\begin{aligned} s_i &= e_{iv} + \Lambda e_{ip} \\ e_{ip} &= \eta_i - \eta_{ir} \\ e_{iv} &= \nu_i - \nu_{ir} \end{aligned} \tag{3.11}$$

Λ = Positive definite matrix

It will prove next that by choosing the parameter adaptation law as given in (3.12), the closed loop formation control of AUVs achieves the desired trajectory tracking and ensure the stability of the formation of AUVs.

$$\dot{\hat{\alpha}} = -\Gamma Y_i^T s_i \tag{3.12}$$

where Γ is the positive definite symmetric matrix.

The structure of the proposed adaptive control law is shown in Figure 3.3.

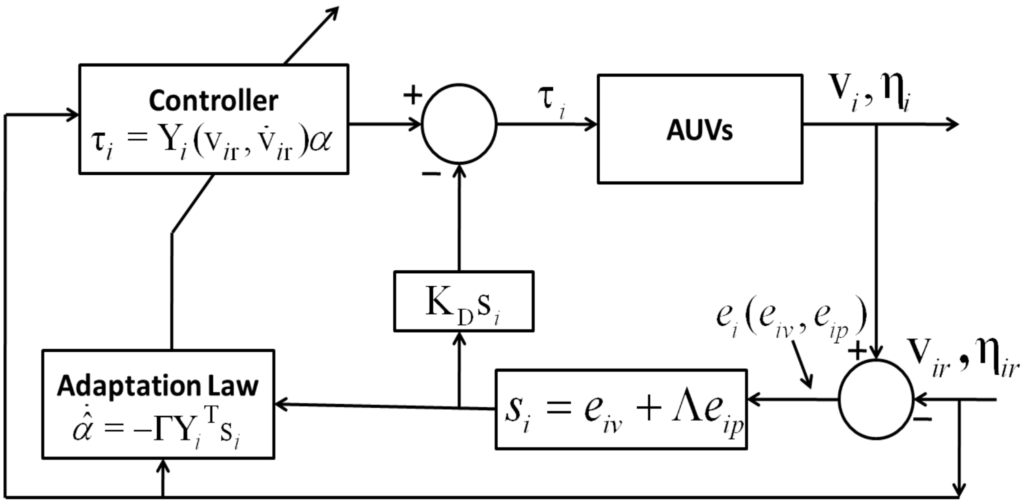


Figure 3.3: Schematic representation of control structure

For mathematical simplicity, considering four DOF of the AUV model, eq. (3.9) can be simplified as

$$M_i \ddot{\eta}_{ir} + D_i(\eta_i, \dot{\eta}_i)\dot{\eta}_{ir} = \tau_i \tag{3.13}$$

or

$$\begin{bmatrix} m_{i11} & 0 & 0 & 0 \\ 0 & m_{i22} & 0 & 0 \\ 0 & 0 & m_{i33} & 0 \\ 0 & 0 & 0 & m_{i44} \end{bmatrix} \begin{bmatrix} \dot{u}_{ir} \\ \dot{v}_{ir} \\ \dot{w}_{ir} \\ \dot{r}_{ir} \end{bmatrix} + \begin{bmatrix} d_{i11} & 0 & 0 & 0 \\ 0 & d_{i22} & 0 & 0 \\ 0 & 0 & d_{i33} & 0 \\ 0 & 0 & 0 & d_{i44} \end{bmatrix} \begin{bmatrix} u_{ir}|u_{ir}| \\ v_{ir}|v_{ir}| \\ w_{ir}|w_{ir}| \\ r_{ir}|r_{ir}| \end{bmatrix} = \begin{bmatrix} \tau_{ix} \\ \tau_{iy} \\ \tau_{iz} \\ \tau_{ir} \end{bmatrix} \tag{3.14}$$

Define the reference trajectory that the AUVs used to follow as given by

$$\dot{x}_{ir} = u_{ir}, \dot{y}_{ir} = v_{ir}, \dot{z}_{ir} = w_{ir}, \dot{\psi}_{ir} = r_{ir} \tag{3.15}$$

Eq. (3.14) can be represented implicitly in the following form as

$$\begin{aligned} m_{i11}\dot{u}_{ir} + d_{11}|u_{ir}|u_{ir} &= \tau_{ix} \\ m_{i22}\dot{v}_{ir} + d_{22}|v_{ir}|v_{ir} &= \tau_{iy} \\ m_{i33}\dot{w}_{ir} + d_{33}|w_{ir}|w_{r} &= \tau_{iz} \\ m_{i44}\dot{r}_{ir} + d_{44}|r_{ir}|r_{ir} &= \tau_{ir} \end{aligned} \tag{3.16}$$

Eq. (3.16) can be expressed in matrix form as

$$Y_i(\nu_{ir}, \dot{\nu}_{ir})\alpha = \tau_i \tag{3.17}$$

where

$$\nu_r = [u_r, v_r, w_r, r_r]^T \tag{3.18}$$

$$Y_i(\nu_{ir}, \dot{\nu}_{ir}) = \begin{bmatrix} \dot{u}_{ir} & |u_{ir}|u_{ir} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \dot{v}_{ir} & |v_{ir}|v_{ir} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dot{w}_{ir} & |w_{ir}|w_{ir} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \dot{r}_{ir} & |r_{ir}|r_{ir} \end{bmatrix} \tag{3.19}$$

$Y_i(\nu_{ir}, \dot{\nu}_{ir})$ is the regressor matrix.

$$\alpha = [m_{i11}, d_{i11}, m_{i22}, d_{i22}, m_{i33}, d_{i33}, m_{i44}, d_{i44}]^T \tag{3.20}$$

and $\tau_i = \left[\tau_{ix}, \tau_{iy}, \tau_{iz}, \tau_{ir}\right]^T$

The regressor matrix $Y_i(\nu_{ir}, \dot{\nu}_{ir})$ is used to represent the AUV model in a linear parametric form. The derivation of the regressor matrix of a high-DOF is tedious. For that reason here only four DOF is considered. For real-time realization, the regressor matrix is to be computed in every sample cycle. Each parameter of the regressor matrix is computed in each iteration [219].

The stability of the proposed adaptive controller is proved by using Lyapunov stability criterion.

Let $V(t)$ be the Lyapunov candidate function given by

$$V(t) = \frac{1}{2}\left[s_i^T M_i s_i + \tilde{\alpha}^T \Gamma^{-1} \tilde{\alpha} \right] = V_1(t) + V_2(t) \tag{3.21}$$

where

$$V_1(t) = \frac{1}{2} s_i^T M_i s_i \tag{3.22}$$

and

$$V_2(t) = \frac{1}{2} \tilde{\alpha}^T \Gamma^{-1} \tilde{\alpha} \tag{3.23}$$

Taking derivative of (3.21) gives

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) \tag{3.24}$$

$$\dot{V}_1(t) = s_i^T M_i \dot{s}_i + \frac{1}{2} s_i^T \dot{M}_i s_i \tag{3.25}$$

Applying (3.11) and $s_i = \dot{\eta}_i - \dot{\eta}_{ir}$ to (3.25),

one gets

$$\dot{V}_1(t) = s_i^T M_i (\ddot{\eta}_i - \ddot{\eta}_{ir}) + \frac{1}{2} s_i^T \dot{M}_i s_i \tag{3.26}$$

Substituting the value of $M_i(\eta_i)\ddot{\eta}_i$ from (3.5) into (3.26) and solving for $\dot{V}_1(t)$ one obtains,

$$\dot{V}_1(t) = s_i^T \left(\tau_i - C_i \dot{\eta}_i - D_i \dot{\eta}_i - g_i - M_i \ddot{\eta}_{ir} \right) + \frac{1}{2} s_i^T \dot{M}_i s_i \tag{3.27}$$

But $\dot{\eta}_i = s_i + \dot{\eta}_{ir}$

Hence,

$$\dot{V}_1(t) = s_i^T \left(\tau_i C_i - \dot{\eta}_{ir} - D_i \dot{\eta}_{ir} - g_i - M_i \ddot{\eta}_{ir} - (C_i + D_i) s_i \right) + \frac{1}{2} s_i^T \dot{M}_i s_i \tag{3.28}$$

or

$$\dot{V}_1(t) = s_i^T \left(\tau_i - M_i \ddot{\eta}_{ir} - C_i \dot{\eta}_{ir} - D_{ir} \dot{\eta}_i - g_i \right) + \frac{1}{2} s_i^T \left(\dot{M}_i - 2(C_i + D_i) \right) s_i \tag{3.29}$$

But for an AUV dynamic equation, it can be easily verified that $\dot{M}_i - 2(C_i + D_i)$ is the skew-symmetric matrix [220]. Using system dynamics, (3.29) reduces to

$$\dot{V}_1(t) = s_i^T \left(\tau_i - M_i\ddot{\eta}_{ir} - C_i\dot{\eta}_{ir} - D_{ir}\dot{\eta}_i - g_i\right) \tag{3.30}$$

Now second term of (3.24) is rewritten as

$$V_2(t) = \frac{1}{2}\tilde{\alpha}^T \Gamma^{-1}\tilde{\alpha} \tag{3.31}$$

with $\tilde{\alpha} = \hat{\alpha} - \alpha$, $\dot{\tilde{\alpha}} = \dot{\hat{\alpha}}$, as α is a constant definite vector

The first derivative of (3.31) can be obtained as

$$\dot{V}_2(t) = \dot{\tilde{\alpha}}^T \Gamma^{-1}\tilde{\alpha} \tag{3.32}$$

Therefore

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) = s_i^T \left(\tau_i - M_i\ddot{\eta}_{ir} - C\dot{\eta}_{ir} - D_{ir}\dot{\eta}_i - g_i\right) + \dot{\tilde{\alpha}}^T \Gamma^{-1}\tilde{\alpha} \tag{3.33}$$

Using (3.9) in (3.33) one gets

$$\dot{V}(t) = s_i^T \left(\tau_i - Y_i\tilde{\alpha}\right) + \dot{\tilde{\alpha}}^T \Gamma^{-1}\tilde{\alpha} \tag{3.34}$$

Taking the controller input $\tau_i = Y_i\hat{\alpha} - K_D s_i$ and solving (3.34), one obtains,

$$\dot{V}(t) = s_i^T Y_i\tilde{\alpha} - s_i^T K_D s_i + \dot{\tilde{\alpha}}^T \Gamma^{-1}\tilde{\alpha} \tag{3.35}$$

Substituting $\dot{\hat{\alpha}} = -\Gamma Y_i^T s_i$, $\dot{V}(t)$ becomes

$$\dot{V}(t) = -s_i^T K_D s_i \le 0 \tag{3.36}$$

Eq. (3.36) satisfies the Lyapunov stability criterion for the group of AUVs system to be stable. Hence the proposed adaptive controller given by (3.36) is stable.

3.5 Results and Discussions

The distributed formation control of multiple AUVs is the combination of path following control of individual AUVs [156]. Each AUV is assigned the desired circular path and the formation goal is fulfilled by combining the tasks of all AUVs in the group as a whole. There are two different desired paths considered for investigating the performance of the developed controller. These are circular and spiral paths.

3.5.1 Circular Path

Let a reference circular path in space for first AUV described as follows

$$\begin{aligned} x_{1r} &= 10\sin(0.01t) \\ y_{1r} &= 10\cos(0.01t) \\ z_{1r} &= 10, \quad \psi_{1r} = \frac{\pi}{3} \end{aligned} \tag{3.37}$$

Similarly the reference paths for second and third AUVs are described as

$$\begin{aligned} x_{2r} &= 20\sin(0.01t) & x_{3r} &= 30\sin(0.01t) \\ y_{2r} &= 20\cos(0.01t) & y_{3r} &= 30\cos(0.01t) \\ z_{2r} &= 10, \quad \psi_{2r} = \frac{\pi}{3} & z_{3r} &= 10, \quad \psi_{3r} = \frac{\pi}{3} \end{aligned} \tag{3.38}$$

The first and second derivatives of position and velocity terms used in simulation are

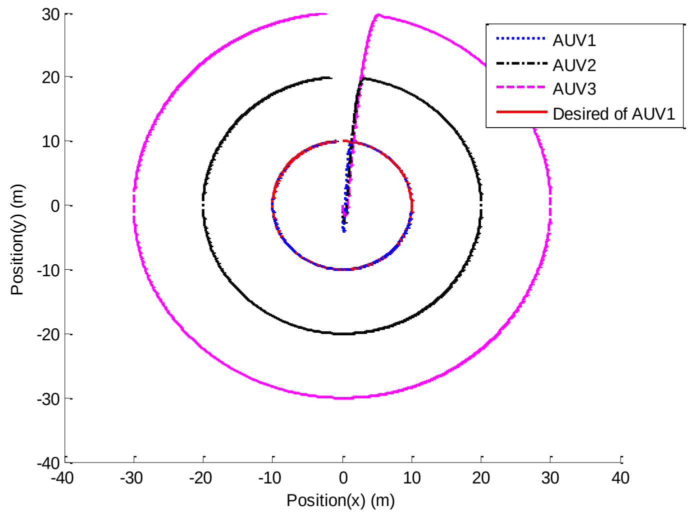
$$\begin{aligned} &\dot{x}_{1r}, \dot{y}_{1r}, \dot{z}_{1r}, \dot{\psi}_{1r}, \ddot{x}_{1r}, \ddot{y}_{1r}, \ddot{z}_{1r}, \ddot{\psi}_{1r} \\ &\dot{x}_{2r}, \dot{y}_{2r}, \dot{z}_{2r}, \dot{\psi}_{2r}, \ddot{x}_{2r}, \ddot{y}_{2r}, \ddot{z}_{2r}, \ddot{\psi}_{2r} \\ &\dot{x}_{3r}, \dot{y}_{3r}, \dot{z}_{3r}, \dot{\psi}_{3r}, \ddot{x}_{3r}, \ddot{y}_{3r}, \ddot{z}_{3r}, \ddot{\psi}_{3r} \end{aligned}$$

The other parameters which are necessary for simulation used from reference [221] are given in Table 3.1.

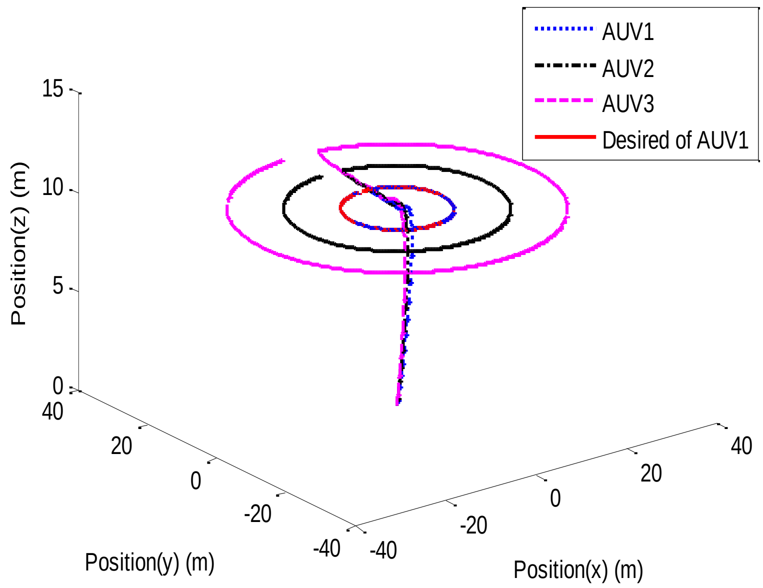
Table 3.1: Parameters of AUV used for simulation

Mass (kg)	Damping coefficients (kg/s)	Others parameters
m_{i11}=99.00 m_{i22}=108.50 m_{i33}=126.50 m_{i44}=29.10	d_{i11} = 10+227.18|u| d_{i22} = 405.42|v| d_{i33} = 10+227.18|w| d_{i44}=1.603+12.937|r	$\mathrm{K_D} = diag\{10, 20, 50, 10\}$ $\Lambda = diag\{50, 10, 20, 20\}$ $\Gamma = diag\{200, 10, 20, 0, 0, 0, 10, 10\}$

The simulation results are presented here.



(a)



(b)

Figure 3.4: Circular trajectory tracking and formation of a group of three AUVs.(a) in plane (b) in space

Figure 3.4 shows a group of three AUVs is tracking the desired circular paths given in (3.37) and (3.38). From this figure, it is seen that the AUVs track the desired trajectory.

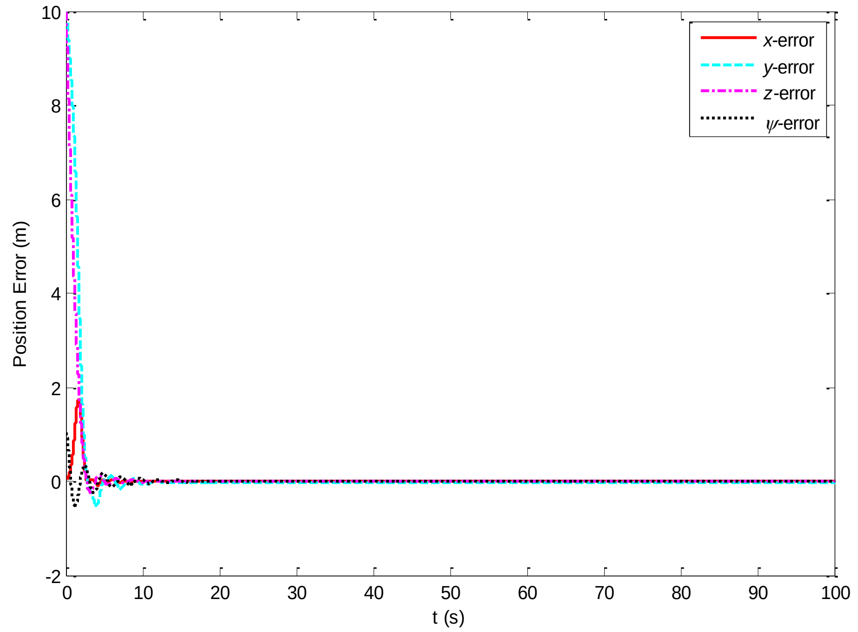


Figure 3.5: Position errors of AUV1

Figure 3.5 shows the position errors found comparing the reference and actual positions as well as orientation of AUV1 in the earth fixed inertial frame. From this figure, it is observed that as the position errors converge to zero. Thus, the proposed controller is effective in providing good tracking of positions.

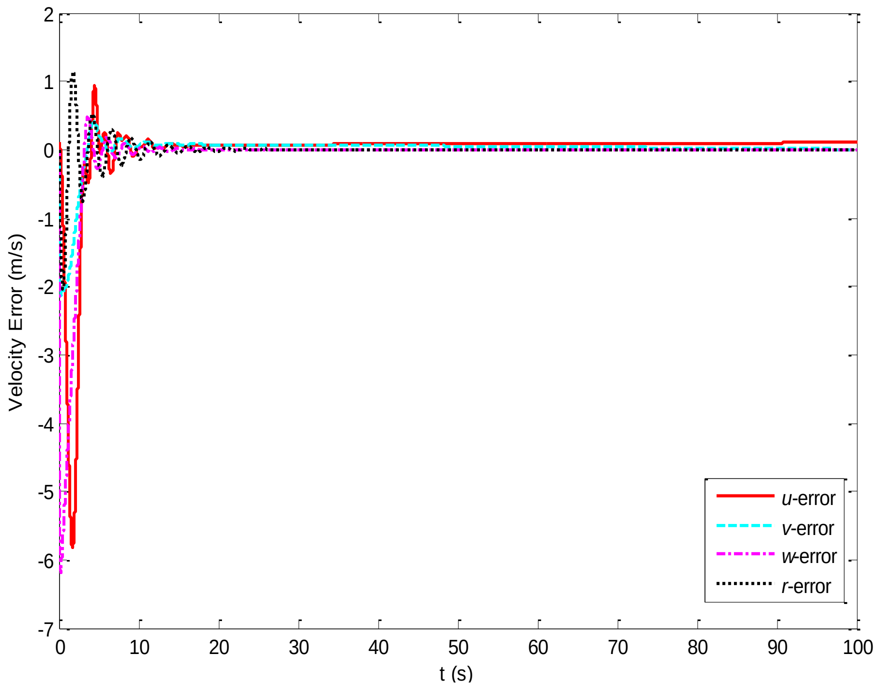


Figure 3.6: Velocity errors of AUV1

Figure 3.6 shows the errors between the desired and actual linear velocities as well as angular velocity of the AUV1 in the body fixed frame. From this figure, it is found that the velocity errors converge to zero.

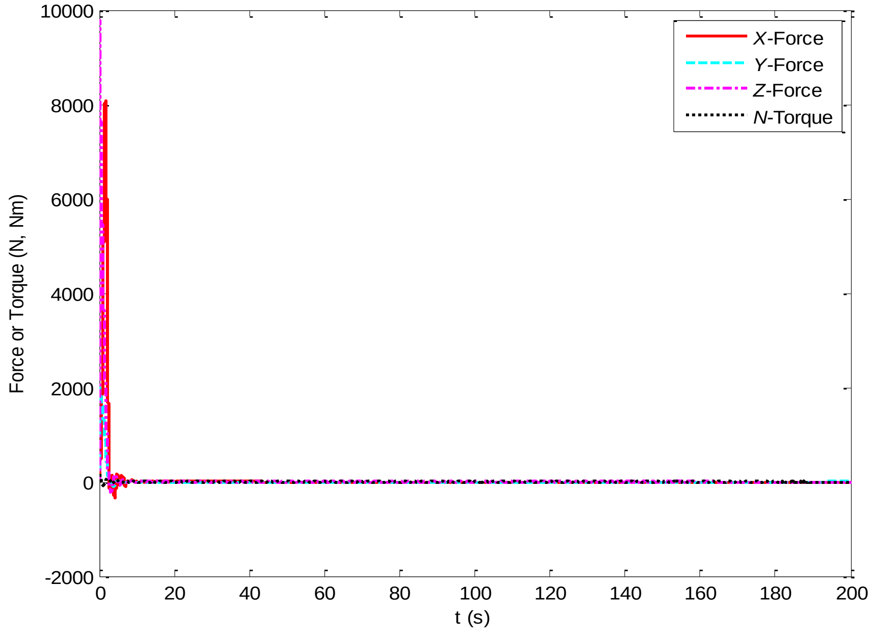
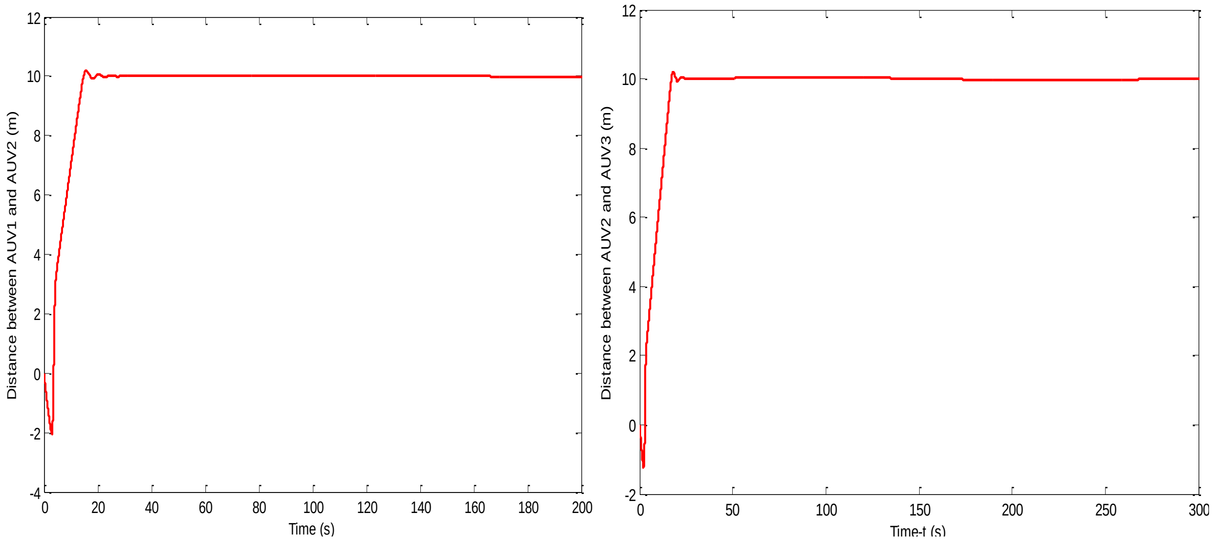


Figure 3.7: Forces and torque of AUV1

Figure 3.7 shows the forces and torque applied to the AUV1 during its motion in formation control. From these results it is observed that at the initial stage the AUVs need some force and torque. But after a definite time when the AUVs reach at their desired track the force and torque reduced to zero. Hence the accelerations reduce to zero, which make the velocities to constant. Thus the controllers able to drive the AUVs along the desired track smoothly.

To avoid collision and uniform motion along the desired trajectories, the AUVs maintain equal distance of separation to accomplish the given task in a group. As the group consists of only three AUVs, the distance between AUV1 and AUV2 is equal to that of the distance between AUV2 and AUV3.



(a) (b)

Figure 3.8: Distance between (a) AUV1 and AUV2 (b) AUV2 and AUV3 during formation in circular path

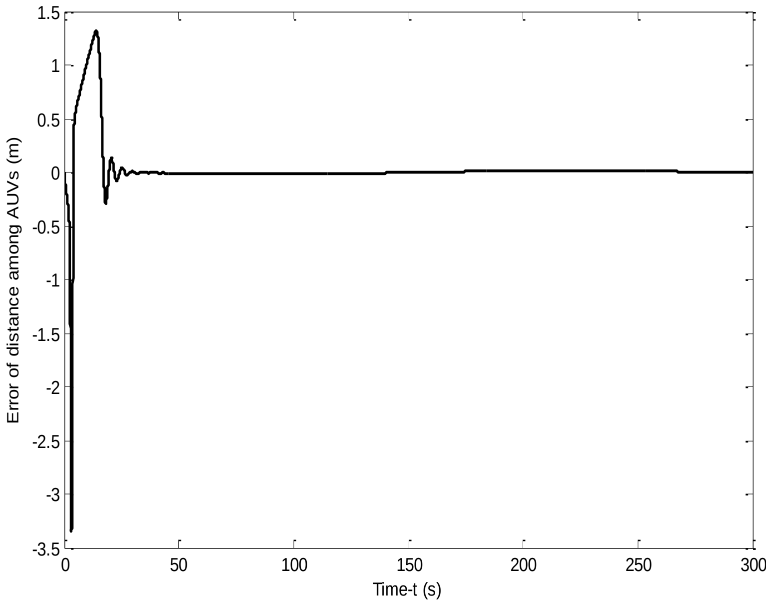


Figure 3.9: Error of distance among AUVs during formation

From Figure 3.8 (a) and Figure 3.8 (b) it is clear that the distance between AUV1 & AUV2 as well as the distance between AUV2 and AUV3 remains constant at 10m during formation along a desired circular path. Figure 3.9 shows the difference of distances between AUV1 & AUV2 and that of AUV2 & AUV3 during formation which is zero. Thus, the error in distances of formation control is zero.

From results, it is clear that, the trajectory tracked by AUV1 coincides with the desired path and the errors become zero. It is also clear that the distances among AUVs remain constant during travelling in the desired trajectories in formation.

3.5.2 Spiral Path

A reference spiral path in space for first AUV is described as follows.

$$\begin{aligned} x_{1r} &= 10\sin(0.01t) \\ y_{1r} &= 10\cos(0.01t) \\ z_{1r} &= t, \;\; \psi_{1r} = \frac{\pi}{3} \end{aligned} \tag{3.39}$$

Similarly the reference paths for second and third AUVs are described as

$$\begin{aligned} x_{2r} &= 20\sin(0.01t) & x_{3r} &= 30\sin(0.01t) \\ y_{2r} &= 20\cos(0.01t) & y_{3r} &= 30\cos(0.01t) \\ z_{2r} &= t, \;\; \psi_{2r} = \frac{\pi}{3} & z_{3r} &= t, \;\; \psi_{3r} = \frac{\pi}{3} \end{aligned} \tag{3.40}$$

The first and second derivatives of position and velocity terms used in simulation are

$$\begin{gathered} \dot{x}_{1r}, \dot{y}_{1r}, \dot{z}_{1r}, \dot{\psi}_{1r}, \ddot{x}_{1r}, \ddot{y}_{1r}, \ddot{z}_{1r}, \ddot{\psi}_{1r} \\ \dot{x}_{2r}, \dot{y}_{2r}, \dot{z}_{2r}, \dot{\psi}_{2r}, \ddot{x}_{2r}, \ddot{y}_{2r}, \ddot{z}_{2r}, \ddot{\psi}_{2r} \\ \dot{x}_{3r}, \dot{y}_{3r}, \dot{z}_{3r}, \dot{\psi}_{3r}, \ddot{x}_{3r}, \ddot{y}_{3r}, \ddot{z}_{3r}, \ddot{\psi}_{3r} \end{gathered}$$

The other parameters which are necessary for simulation are the same as in case I.

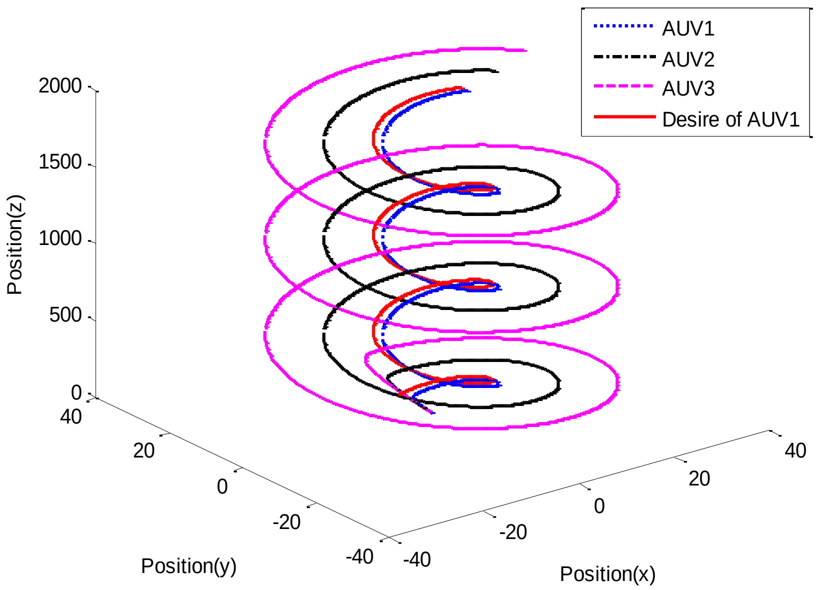


Figure 3.10: Spiral trajectory tracking and formation of a group of three AUVs

Figure 3.10 shows a group of three AUVs are tracking the desired spiral path. From this figure it is seen that the positions of the desired trajectory coincides with that of the actual trajectory.

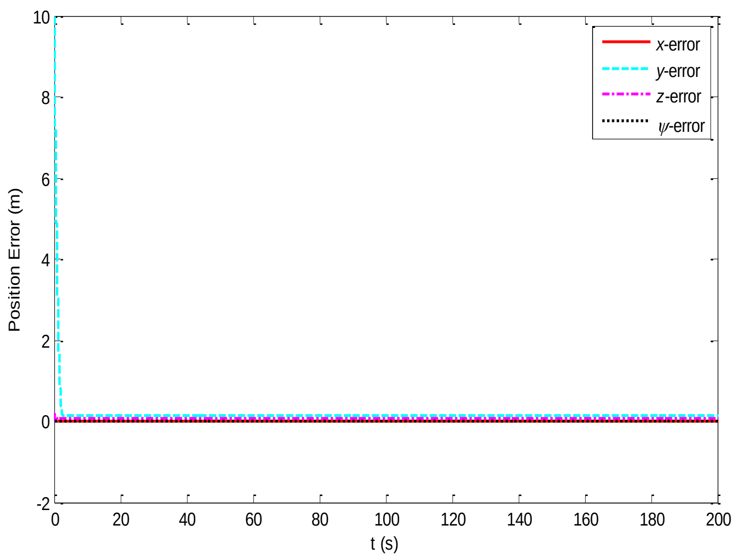


Figure 3.11: Position errors of AUV1

Figure 3.11 shows the position and orientation errors of AUV1 in the earth fixed inertial frame. From this figure, it is seen that the position errors converge to zero. Hence, the AUV1 is able to track the reference spiral path accurately.

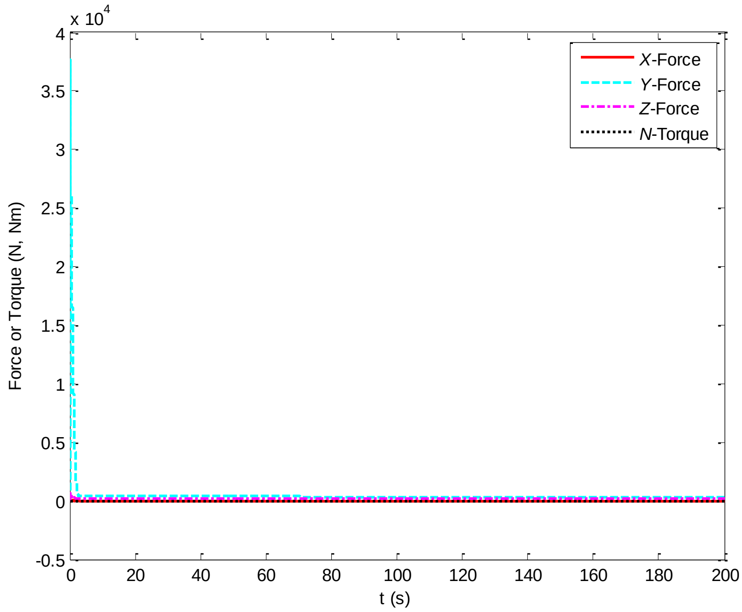
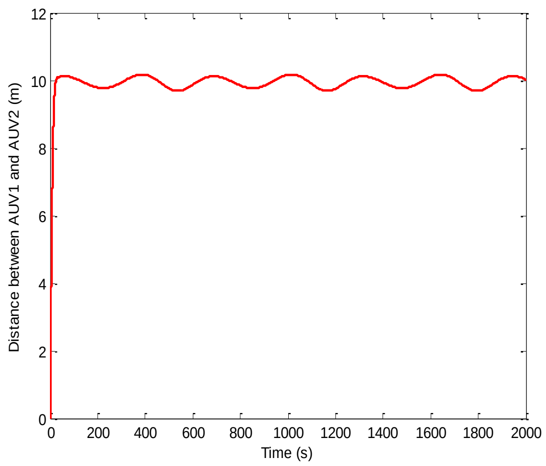
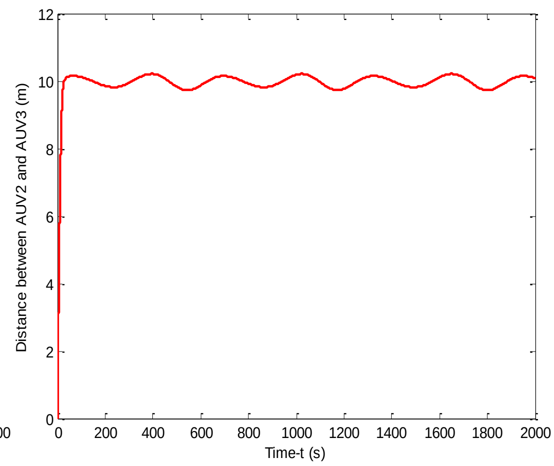


Figure 3.12: Forces and torque of AUV1

Figure 3.12 shows the forces and torque applied to the AUV1 during moving in formation control. The distance between AUV1 and AUV2 is equal to that of the distance between AUV2 and AUV3.



(a)



(b)

Figure 3.13: Distance between (a)AUV1 and AUV2 (b) AUV2 and AUV3 during formation in spiral path

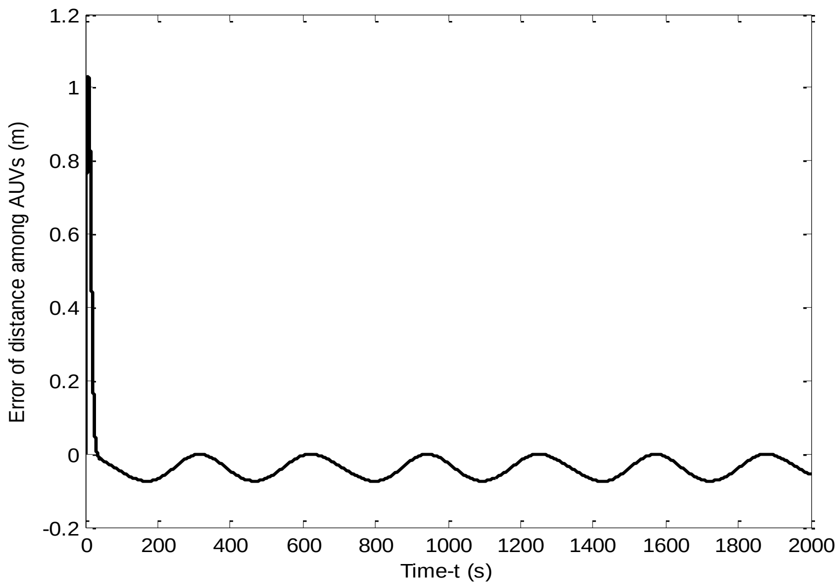


Figure 3.14: Error of distance among AUVs during formation

From Figure 3.13 (a) it is clear that the distance between AUV1 and AUV2 remains constant at 10 m during maintenance of formation moving in the circular path. Figure 3.13 (b) shows the distance between AUV2 and AUV3 also remains constant at 10 m during maintenance of formation moving in the circular path. Figure 3.14 shows the difference of distance between AUV1 and AUV2 and that of AUV2 and AUV3 during formation and is zero. Hence it is observed that all the AUVs maintain equal distance during formation.

3.6 Chapter Summary

The chapter proposed an adaptive formation control algorithm for a group of AUVs considering the uncertainties associated due to hydrodynamic parameters. The proposed adaptive control law is applied to guide individual AUVs to track the desired trajectories where formation control is the combination of the tasks performed by individual AUVs. The stability of the proposed control law is ensured by exploiting the Lyapunov stability criterion. From the simulation results it is observed that, when the adaptive formation control law is applied to a group of AUVs to track the desired trajectories, it exhibits effective cooperative motion control performance.

Chapter 4

Formation Control of AUVs Navigating Towards a Safety Region

This chapter presents the development of a gravity compensation proportional derivative (GCPD) formation control law based on potential functions for a group of AUVs. The objective of this chapter is to develop a control law which steers a group of multiple AUVs towards a safety region without any collision among them during tracking. The safety region is considered as a cylindrical region, which is surrounded by unsafe region. Inter distance dependent potential functions are used to avoid collision among the AUVs. For simulation purpose, a special kind of AUV i.e. Omni Directional Intelligent Navigator (ODIN) is taken into consideration. A group of three AUVs is considered for studying the efficacy of the developed control law. From the obtained results it is found that, this algorithm provides an effective co-operative motion control law for a group of multiple AUVs to steer towards a desired safety cylindrical region.

4.1 Introduction

Out of many research topics in multi robot systems, formation control of the multi AUV system is an important as well interesting topic in the current era. Moving in formation has many advantages over conventional systems, for examples, it can reduce the system cost, increase the robustness and efficiency of the system while providing redundancy, reconfiguration ability and structure flexibility for the system [1]. The formation control of AUVs has applications in commercial solicitations such as in gas and oil industries, in precise surveys or areas where traditional bathymetric surveys are less effective, post-lay pipe surveys, etc.. It has applications in the military field in the map mines area and for protective purposes. Also the formations of AUVs have broad application in research fields such as; to study lakes, ocean and the ocean floor.

In practical cases, the works on formation control of AUVs are more challenging because of their complex nonlinear dynamics. Several methods have been developed to accept these challenges. Some of the methods are reviewed here. In [18], [180], [226] and [254], the control of a group of fleet of ship formation is investigated. The design procedure is based on the manoeuvring and the main goal of control is to maintain its position. A virtual formation reference point (FRP) tracks a predefined path. Here the total problem is divided into two sub problems; (i) a geometric task and (ii) dynamic task. The former fulfils the FRP and hence the formation structure tracks the desired path, i.e. FRP follows the predefined desired path. In [181], [183] and [215], a leader follower formation control algorithm for multiple AUVs is developed. This algorithm employed a variant leader follower formation control strategy. In this type of formation control strategy, to maintain a fixed geometrical formation, the mission way points have to navigate along the desired trajectories. In this method the leader traverses the waypoints using acoustic long baseline (LBL) of the positions. The formation control of marine surface crafts using the Lagrangian mechanics is proposed in [19]. Here the different constraints taken are; (i) distance between members (ii) position constraints (iii) combined constraints (iv) formation average position (v) formation variance. In [20], the formation control law for multiple unmanned under actuated surface vessels is developed by using the sliding mode method. There are two types of controllers, i.e. L-Si and L-L are mentioned. The virtual leader concept is introduced whose trajectory is known to all the members present in formation group. Low level sliding mode controller is designed that are suitable for the surface vessels having an uncertain dynamic model. In [21], a new formation control and obstacle avoidance algorithm based on the potential function is proposed. Here the manoeuvring area is divided into three different parts such as (i) safety area (ii) avoidance area (iii) danger area. The control laws for different areas are developed successively based on the potential function. A formation control law of multiple AUVs using fixed interaction topology criteria is proposed in [178]. Here an adaptive sliding mode control law is designed for controlling the formation group. In [23], the nonlinear formation keeping and mooring control of multiple AUVs in chains form is described. Here the leader follower control strategy of formation control with the integrator back stepping method is used for developing the mooring controller. Formation control of multiple under-actuated surface vessels is proposed in [25] with the use of graph theory. The cooperative control laws are developed not only for a stationary geometric pattern, but also have given tress to the same orientation of all the vessels. The proposed control law is analysed and the stability of the closed loop system with the time delay is presented. A simple PD controller based on Lyapunov's stability analysis is used for formation control of multiple AUVs is explained in [182]. In [174], a control algorithm is proposed to stabilize the formation of AUVs in a circle with time varying centre. A formation control law of multiple AUVs using fuzzy logic based behaviour fusion is discussed in [185]. This work presents a new behavior fusion method using fuzzy logic for coordinating multiple reactive behaviours. A finite consensus based formation control algorithm is developed in [228]. Here the formation control is considered in a limited communication range. Path parameter consensus based formation control of multiple AUVs is developed in the presence of ocean current in [81]. Here the virtual leader follower concept is used for development of control algorithms. In special cases, the formation known as flocking control where the basic idea of goal reaching come from the natural agents such as birds, ants, bacteria etc. [229], [210].

In the above review, only motions of groups of AUVs or mobile robots in different conditions are mentioned. But no literature is found which emphasized on the motion of group of AUVs towards a safety region. As the formation control of multiple AUVs is an important and interesting research topic in the recent era and by motivating in this research topic, a simple potential based PD formation control law is developed in this chapter. The control law is able to steer a group of AUVs towards a safety region without facing any collision among them.

The rest of this chapter is organized as follows. Section 4.4 describes the modelling of an AUV and problem formulation. Potential based PD formation controller for multiple AUVs is developed in Section 4.5. Numerical simulations are reported in Section 4.6 to show the efficacy of the developed controller. Chapter summary is presented in Section 4.7.

4.2 Objectives of the Chapter

- To develop gravity compensated PD formation control law for multiple AUVs based on artificial potential functions related to the tracking area which is divided into different layers.

4.3 Problem Formulation

The objective of this chapter is to design a distributed control law for each AUV such that the multi-AUV system steers to reach a safety region starting from any initial connected topological configuration (Figure 4.1). Here it is considered that the safety region is a cylindrical region and is surrounded by unsafe region. The inner side is also accompanied with unsafe region. So the safety region is only in the form of a ring having some width. Mathematically, it can be represented as,

$$\lim_{t \to \infty} \eta_i = \eta_{i_s} \tag{4.1}$$

where

$\eta_i = [x_i, y_i, z_i, \varphi_i, \theta_i, \psi_i]^T$ is the position of the i^{th} AUV.

$\eta_{i_s} = \left[x_{i_s}, y_{i_s}, z_{i_s}, \varphi_{i_s}, \theta_{i_s}, \psi_{i_s}\right]^T$ is the position of i^{th} AUV in the safety region.

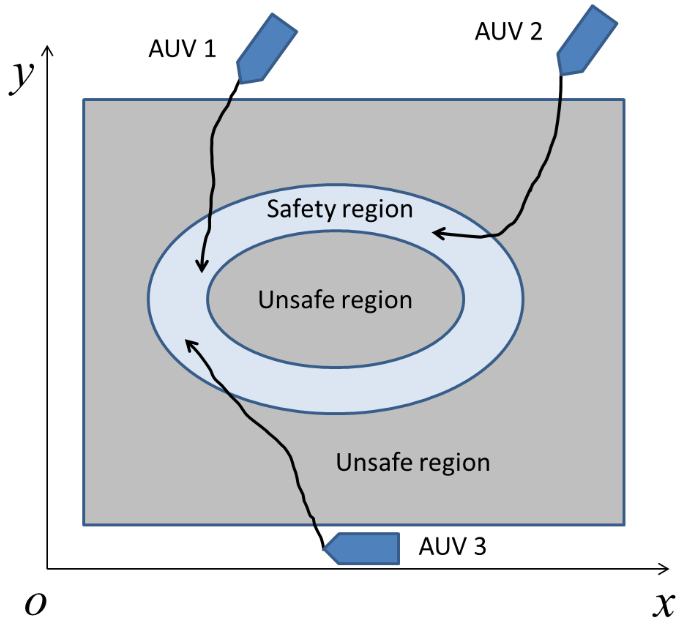


Figure 4.1: Presentation of different regions of AUV tracking area

4.4 Modelling of the AUV

4.4.1 Kinematics of AUV

AUV kinematics and dynamics in six degrees of freedom (DOF) i.e. the vehicle is moving in three dimensional spaces are briefly reviewed here. There are two types of frames of references are considered, i.e. body fixed frame of reference $\{B\}$ and earth fixed frame of reference which is known inertial frame of reference $\{I\}$, (Figure 4.2). .

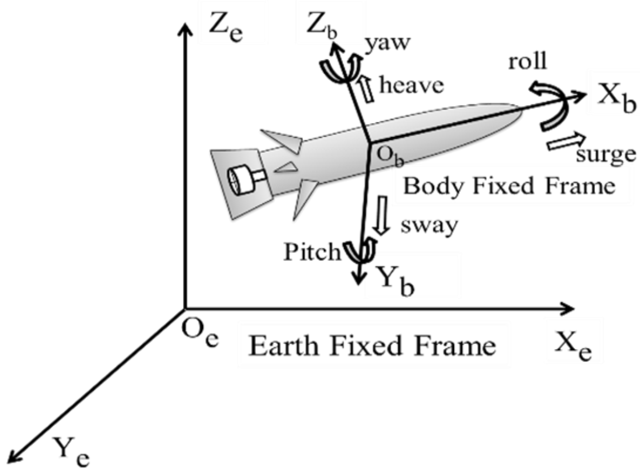


Figure 4.2: Schematic presentation of an AUV in six DOF

The motion of i^{th} AUV in six degrees of freedom (DOF) can be achieved by referring equation (3.3) through (3.8) and presented in a different manner as follows.

The nonlinear dynamical and kinematic equations of motion can be expressed as

$$\begin{aligned} \dot{\eta}_i &= J_i(\eta_i)\nu_i \\ M_i\dot{\nu}_i + C_i(\nu_i)\nu_i + D_i(\nu_i)\nu_i + g_i(\eta_i) &= \tau_i \end{aligned} \tag{4.2}$$

where $M_i(\eta_i)$ is the inertia matrix including added mass, $C_i(\eta_i,\dot{\eta}_i)$ is the matrix of Coriolis and centripetal terms including added mass. $D_i(\eta_i,\dot{\eta}_i)$ denotes hydrodynamic damping and lift matrix and $g_i(\eta_i)$ is the vector of gravitational forces and moments. $J_i(\eta_i)$ is velocity transformation matrix between the AUV and earth fixed frames.

4.5 Development of Control law

In this section we present the development of a PD controller to steer the multiple AUVs to the desired safe region. It is considered that the desired safety area is in the form of a flat ring which is formed by two concentric cylindrical regions having dimensions as follows. [x_o, y_o, z_o] is the coordinate of the position of the centre of the concentric cylinders. R and r are the radii of the outer and inner cylindrical regions respectively. d is the thickness of the desired ring. h is the height of the cylindrical region. Form Figure 4.3, $d = R - r$.

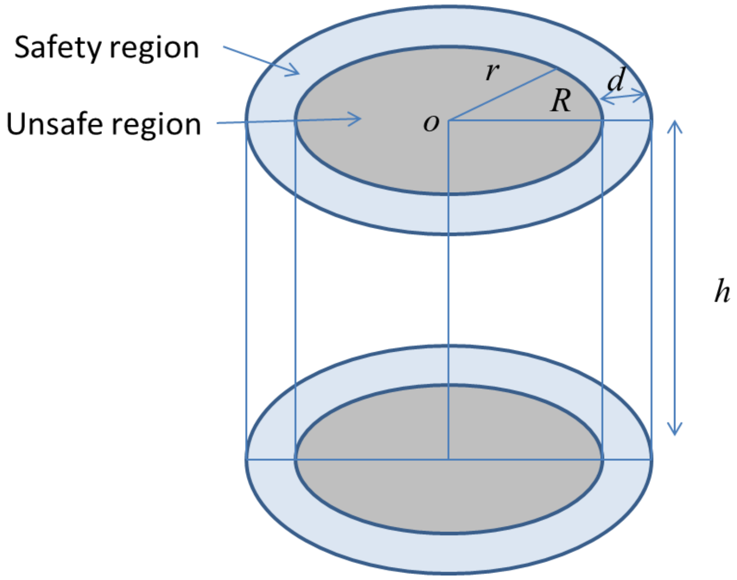


Figure 4.3: Schematic presentation of different regions of desired region

The shape function of the desired region is defined as [228],

$$\begin{aligned} f_{s1}(\Delta\eta_{io1}) &= r^2 - (x_i - x_o)^2 - (y_i - y_o)^2 \\ f_{s2}(\Delta\eta_{io2}) &= (x_i - x_o)^2 + (y_i - y_o)^2 - R^2 \\ f_{s3}(\Delta\eta_{io3}) &= (Z_i - z_o)^2 - \left(\frac{h}{2}\right)^2 \end{aligned} \tag{4.3}$$

Hence the shape function vector for the desired region may be presented as

$$f_s = [\, f_{s1}, f_{s2}, f_{s3}]^T \tag{4.4}$$

The potential energy function of the desired region is described by the inequality function given below.

$$E_{s_i}(\Delta\eta_{iol}) = \sum_{l=1}^{3} E_{s_l}(\Delta\eta_{iol}) \tag{4.5}$$

where

$$E_{s_l}(\Delta\eta_{iol}) = \frac{k_l}{2}[\max(0, f_{s_l}(\Delta\eta_{iol}))]^2 \tag{4.6}$$

Partial differential of this potential energy presented in (4.6) is given as,

$$\left(\frac{\partial E_{s_l}(\Delta\eta_{iol})}{\partial\eta_i}\right)^T = \sum_{l=1}^{3} k_l \max(0, f_{s_l}(\Delta\eta_{iol})) \times \left(\frac{\partial E_{s_i}(\Delta\eta_{iol})}{\partial\Delta\eta_{iol}}\right) = \Delta\varepsilon_i \ (11)$$

Here $\Delta\varepsilon_i$ denote the region error of the desired area.

Collision Avoidance:

In this section the method of avoidance of collision among AUVs is explained. It is considered that each AUV is surrounded by multiple spherical layers in the surrounding as shown in Figure 4.4. Here $r_1 > r_2 > \cdots > r_l$ i.e the outer layer is the largest layer and gradually decreases towards the inner layers.

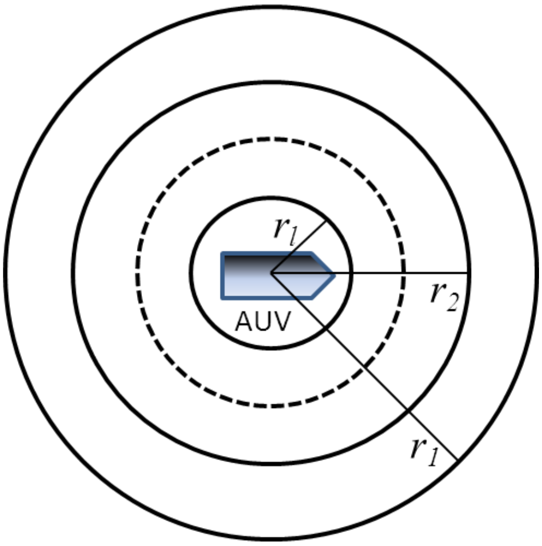


Figure 4.4: Representation of different layers around AUV

Multilayer concept is used to develop the collision avoidance function where the gain of the layers gradually increases from the outermost layer to the inner layer. If the outer layer is not able to provide the sufficient repulsive force to avoid collision, then the consecutive inner layer comes to play the role of repulsion. In this manner the inner layers of the multilayer system activate gradually to avoid the collision among the AUVs. This collision avoidance function between i^{th} and j^{th} AUV are presented here as follows. i = 1, 2, 3, …n, j = 1, 2, 3, …n and $i \neq j$. n is the number of AUVs considered in the group for formation.

$$\begin{aligned} s_{1ij}(\Delta\eta_{ij}) &= r_1^2 - \|\Delta\eta_{ij}\|^2 \\ s_{2ij}(\Delta\eta_{ij}) &= r_2^2 - \|\Delta\eta_{ij}\|^2 \\ &\vdots \\ s_{lij}(\Delta\eta_{ij}) &= r_l^2 - \|\Delta\eta_{ij}\|^2 \end{aligned} \tag{4.7}$$

where $\Delta\eta_{ij} = \eta_i - \eta_j$, $s_{1ij}, s_{2ij}, \cdots s_{lij}$ are the layer functions for 1st (outermost), 2nd and l^{th} (innermost) layer respectively. Potential energies for these layers are presented as follows.

$$\begin{aligned} P_{1ij}(\Delta\eta_{ij}) &= \frac{k_{1ij}}{2}[\max(0, s_{1ij}(\Delta\eta_{ij}))]^2 \\ P_{2ij}(\Delta\eta_{ij}) &= \frac{k_{2ij}}{2}[\max(0, s_{2ij}(\Delta\eta_{ij}))]^2 \\ &\vdots \\ P_{lij}(\Delta\eta_{ij}) &= \frac{k_{lij}}{2}[\max(0, s_{lij}(\Delta\eta_{ij}))]^2 \end{aligned} \tag{4.8}$$

where $k_{1ij}, k_{2ij}, \ldots k_{lij}$ are positive constants.

So the total potential energy between i^{th} and j^{th} AUV required for collision avoidance is given by

$$P_{ij}(\Delta\eta_{ij}) = \sum_{k=1}^{l} E_{kij}(\Delta\eta_{ij}) \tag{4.9}$$

Hence the required interactive repulsive force f_{ij} between i^{th} and j^{th} AUV may be presented by

$$f_{ij} = \left(\frac{\partial P_{ij}(\Delta\eta_{ij})}{\partial\Delta\eta_{ij}}\right)^T = \sum_{u=1}^{l} k_{uij} \max(0, s_{uij}(\Delta\eta_{ij})) \times \left(\frac{\partial s_{uij}(\Delta\eta_{ij})}{\partial\Delta\eta_{ij}}\right) \tag{4.10}$$

As the interactive repulsive force between i^{th} and j^{th} AUV is bidirectional, so $f_{ij} = -f_{ji}$. Hence the total collision avoidance f_c force acting upon i^{th} AUV due to its neighbour AUVs is given by

$$f_{ci} = \sum_{j=n} f_{ij}\ ,\ i \neq j \tag{4.11}$$

For developing controller let us define a quantity function Δf which is the submission of forces due to the region and due to collision avoidance found in (4.7) and (4.12) such that,

$$\Delta f_i = \alpha_i f_{ci} + \beta_i \Delta\varepsilon_i \tag{4.12}$$

where α_i and β_i are positive constants.

The GCPD control law for driving i^{th} AUV in the group may be expressed as follows

$$\tau_i = -J_i^T(\eta_i) K_{pi} \Delta f_i - J_i^T(\eta_i) K_{vi} J_i^T \nu_i + g_i(\eta_i) \tag{4.13}$$

where $K_{pi} = k_p I_{6\times 6}$, k_p is a positive constant, $K_{vi} \in R^{6\times 6}$. The efficacy of this PD controller is verified from the simulation results presented in Section 4.6.

4.6 Results and Discussions

Simulation of formation control of multiple AUVs is carried out considering a group of three AUVs. Here a special type of AUVs taken is Omni Directional Intelligent Navigators (ODIN). The parameter of this AUV is mentioned below.

The buoyancy and gravitational factor present in the dynamic of the i^{th} AUV in six DOF is presented as follows [227], [228].

$$g_i(\eta_i) = \begin{bmatrix} \left(m_i g - \frac{4}{3}\pi r_i^3 \rho g \right) \sin\theta_i \\ -\left(m_i g - \frac{4}{3}\pi r_i^3 \rho g \right) \cos\theta_i \sin\phi_i \\ -\left(m_i g - \frac{4}{3}\pi r_i^3 \rho g \right) \cos\theta_i \cos\phi_i \\ zG_i m_i g \cos\theta_i \sin\phi \\ zG_i m_i g \sin\theta_i \\ 0 \end{bmatrix} \tag{4.14}$$

For simulation purposes the numerical values are considered as mentioned in Table 4.1.

Table 4.1: Parameters of ODIN

Parameter	Symbol	Numerical value
Mass of ODIN	m_i	125 kg
Radious of ODIN	r_i	0.3 m
Distance of CM from CG	zG_i	0.05 m
Density of water	ρ	1000 kg/m^3
Gravitational constant	g	9.81 m/s^2

The desired region cylinder parameters are r = 0.3 m, R = 2.5 m, h = 1 m and placed at (x_o, y_o, z_o) = (10, 10, 10). The radii of the two layers of the AUV for collision avoidance are r_1 = 1 m, r_1 = 0.5 m. The other parameters for simulation are; $K_{vi} = diag\{30, 30, 30, 30, 30, 30\}$, $k_p = 55$, $k_l = 0.1$, $\alpha_i = 1$ and $\beta_i = 2$. The simulation results are presented next.

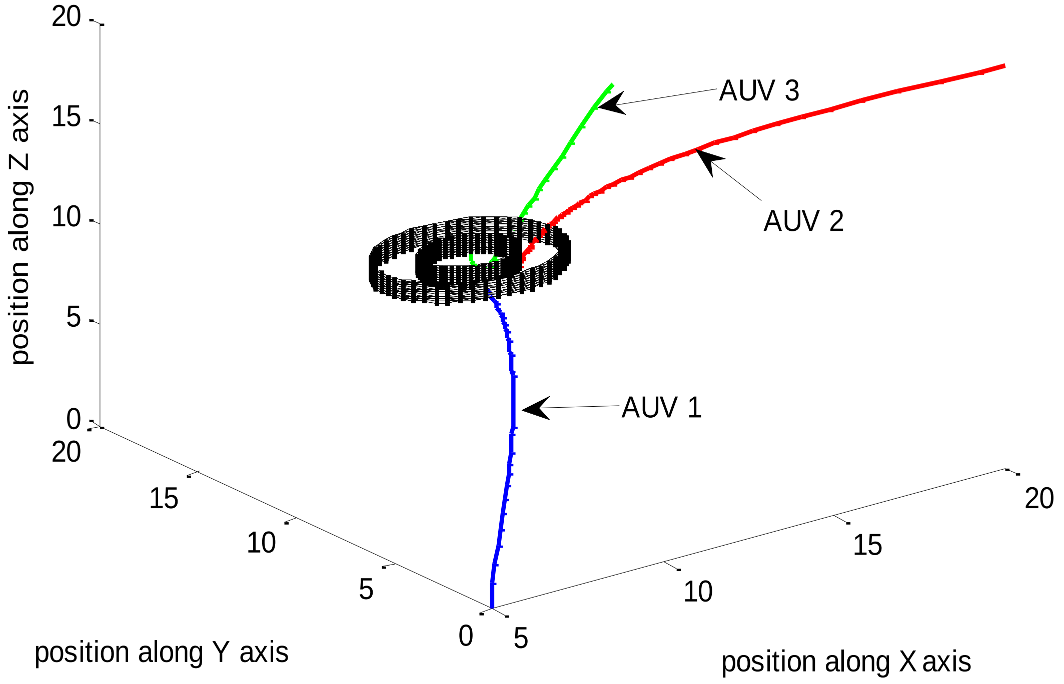


Figure 4.5: Three AUVs are steering towards the safety cylindrical region (side view)

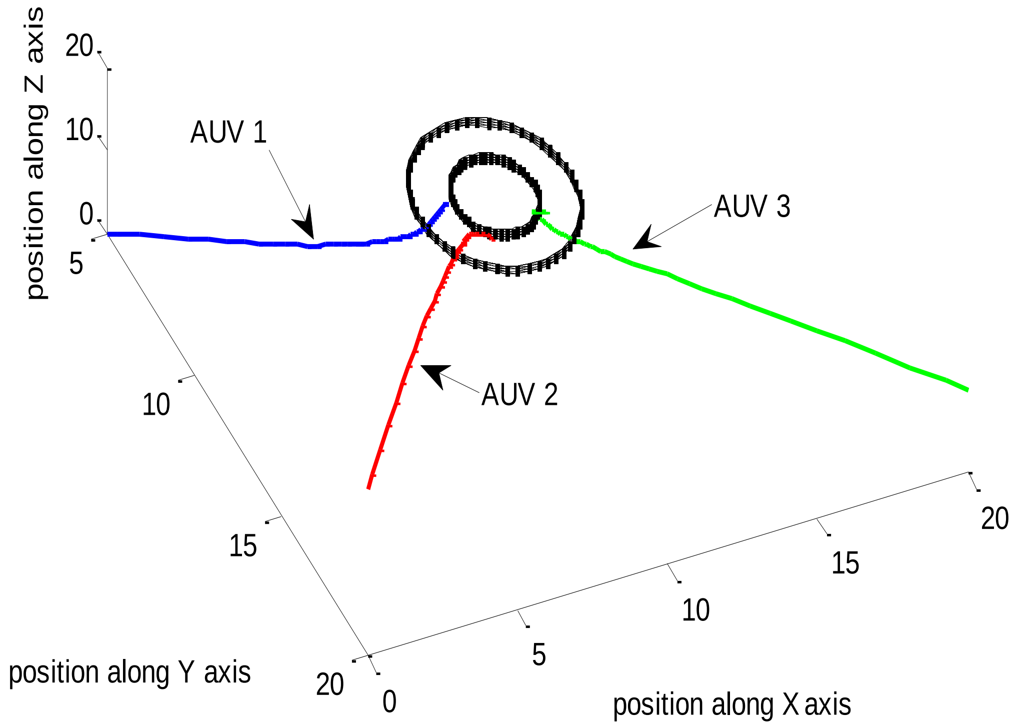


Figure 4.6: Three AUVs are steering towards the safety cylindrical region (top view)

Figure 4.5 and Figure 4.6 show the simulation results of the formation control of three AUVs. From these figures it is observed that the AUVs steer towards a cylindrical safety region after passing the unsafe region. During their motion, they do not collide with each other. So the developed potential based GCPD controller is able to drive the group of AUVs to travel to the desired region.

4.7 Chapter Summary

Potential function based GCPD formation control is developed for a group of three AUVs. The shape functions of the different regions of the environment are established using the potential functions accompanied with individual regions. Potential energy based mathematical functions are used to avoid the collision among AUVs. The efficacy and accuracy of the developed control law are verified through simulation of three AUVs considering in a group. From the results obtained, it is observed that the AUVs are able to steer to the desired safety region without collision.

Chapter 5

Flocking Control of AUVs based on Mathematical Potential Functions

In this chapter a flocking control algorithm for a group of multiple AUVs using the leader-follower concept both considering without communication constraints and with communication constraints. Leaders have global knowledge of desired trajectory and other AUVs are chosen as followers, which are assumed to be equipped with sonar sensors to find the positions of neighbour AUVs without any knowledge of the desired path. The leader AUVs are expected to track the desired trajectory with the application of the developed flocking controller and the follower AUVs are attracted towards the leader AUVs so that the whole group tracks the desired trajectory. The flocking controller is developed by deploying a virtual point known as flocking centre and its position is estimated by applying the consensus algorithm. A flocking controller for controlling leader AUV and its flock mates to flock in desired trajectory is developed using artificial potential functions (APF). The APF is basically used to avoid the inter AUV collision and collision between AUVs with obstacles. The efficacy of the developed controller is verified through simulation studies considering four AUVs flocking in a group.

5.1 Introduction

Figure 5.1: Flocking of different animals in nature

Flocking is the flying behaviour of a group (flock) of birds. This is the overall behaviour of a group of self-repellent and cohesive agents. This is also a collective behaviour of living beings such as schooling of fishes, flocks of birds, grouping of insects, colonies of bacteria and herds of animals etc. (Figure 5.1). In nature, animals achieve these grouping properties in order to increase the chance of getting foods, saving energy, to escape from enemies and to confuse the other animals present in higher level in the food-chain and food-net. The basic rule of the flocking model includes separation (short range repulsion), alignment (same orientation) and cohesion (long range attraction).

First mathematical modelling and simulation work on flocking has been proposed by Reynolds [10]. Both flocking of multiple agents and obstacle avoidance are considered in theoretical and computational distributed flocking manner [209]. The flocking control proposed by Saber et al., in [209] is modified by Su et. al., in [253] where a fraction of uniform agents is used to drive multiple agents and the velocities of agents are not constant and are equal to that of the desired velocities. The virtual leader concept is employed for flocking of multiple agents in [249]. The velocity is considered to be constant here. Su et.al. proposed flocking control algorithm for a group of agents using multiple virtual leaders [234]. Here the total group is divided into many subgroups having an individual virtual leader and these subgroups possess the velocity equal to that of the virtual leader. The application of Voronoi diagram and Central Voronoi Tessellation (CVT) is used in distributed flocking control of mobile agents [231]. CVT is extended to dynamic environments. The automatic flocking behavior of a mobile robot is achieved by using CVT area coverage. Based on relative distance between agents, a fuzzy logic approached flocking controller is developed by Wang. et. al. [232]. Here one virtual quantity called force function is used to control the velocities of the agents. Tanner et al. explained decentralized flocking algorithms for both fixed and switching network topology [233]. The stability analyses of a group of mobile agents are accomplished considering velocities and inter agent distances as variable parameters. A distributed flocking algorithm has been proposed where the information exchange between mobile agents is modelled by communication topology [235]. Here the sub-graph containing all the vertices is proved to be connected as long as the initial communication topology is connected. A distributed control law is developed for the flocking motion preserving all the vertices in the sub-graph. Flocking of a class of multi agent system with switching topology is considered in noisy environments by Li et al. [256]. Here it is proved that, contaminating the information between flock-mates with environmental noise, the flocking is pursued if the gradient of the noise is bounded and the graph is jointly connected. Sharma et al. developed a flocking control of non-holonomic car like vehicles in a constrained environment [236]. Here the centralized motion planner is proposed for manoeuvring the flock-mates. A Lyapunov stability based control structure is developed for avoiding obstacles and the leader follower framework is used to purse the flocking of multiple vehicles. Assuming that every agent can accept a certain number of flock-mates, a flocking algorithm for multiple agents is developed by Luo et al. [237]. For designing a controller, there are two kinds of potential functions are used depending upon the nature of agents, whether they are moving towards the same agent or different. Under graph theory, connectivity constraint, a distributed flocking algorithm for multiple mobile agents has been investigated by Yutian et al [259] and for multiple AUVs in [210]. Hybrid artificial potential function and graph connectivity problem is considered for development of controller for collision and obstacle avoidance. A control law for the flocking of multiple AUVs based on artificial fuzzy potential function is analysed in in [229]. Here the inter distance between the AUVs is used as fuzzy input function and the repulsive potential function is observed as the output of the fuzzy inference system. La et. al. developed a flocking algorithms for multiple agents in noisy environments in [238]. Based on this algorithm, all agents are connected and the information exchange between flock-mates took place through the connected graph topology. Lu et al. presented some practical control protocols for flocking problems in [257]. Here diverse ranges of control protocols for different consensus problems are unified to a single problem assuming unknown virtual linear velocity. The collision and obstacle avoidance is guaranteed by this algorithm. Flocking control of multiple mobile agents using measured velocity is considered by Wen et al. in [249]. The velocities of dynamic agents are converse with that of consensus velocities ensuring collision avoidance among the agents. A nonlinear dynamic virtual leader concept is also explained here. Considering the connectivity of the graph containing all the vertices and using the second smallest eigenvalue of the concern Laplacian, the flocking algorithm for multiple agents is developed in [258]. Zhoua et al. accomplished the flocking of multiple agents using a pseudo leader and virtual force mechanism. This mechanism is applied to an uncompleted or switching neighbour graph [250]. Flocking of a group of multiple micro-particles using velocity saturation method are proposed in [243].

In the existing literature discussed above, flocking of mobile agents and car like vehicles are reported. But a few works on flocking control of multiple AUVs have been reported in literature. Applications of AUVs in different important fields such as military, oil industry, monitoring of oceanic shallow-water regions and environmental surveying, etc. [36], [105] created enormously interesting in pursuing research in this area. A group of AUVs may be needed to follow a predefined trajectory while maintaining a desired spatial pattern. Moving in flocking may have many advantages such as reduction of the system cost, increased robustness and efficiency of the system while providing redundancy, reconfiguration ability and structure flexibility for the system etc.. Hence flocking control of a group of AUVs is an important research topic.

The contributions of this chapter are as follows. A flocking control algorithm has been developed for keeping multiple AUVs together in groups by using leader-follower control strategy and consensus protocol. The artificial potential function concept is used to design the controller to avoid obstacles appearing on the desired path. Here, assuming that all AUVs are well communicated with their flock-mates and it is assumed that the states of AUVs to be available.

The leader AUVs have the prior knowledge about the desired trajectory to be followed, but the follower vehicles which follow the leader have no knowledge of desired trajectories. All vehicles have the knowledge about the flocking centre which can be calculated using the consensus algorithm. Every vehicle maintains a certain distance from the flocking centre. All the vehicles would attach to that of flocking centre and follow it when travel in the desired trajectory. Bounded artificial potential functions such as mathematical separation potential functions are used for development of controller for the leader as well as follower AUVs. Hence all the AUVs would be able to move in a group together avoiding collision among them as well as avoiding collision with obstacles intersecting the desired trajectories of the AUVs.

The rest of the chapter is organized as follows. Chapter objective is presented in Section 5.2. Flocking control problem is formulated in Section 5.3. Section 5.4 describes about the modelling of an AUV and problem formulation. Fundamentals of graph theory and consensus algorithm for multiple AUVs are presented in Section 5.5. Mathematical RPF analyses are presented in Section 5.6. Numerical simulations are reported in Section 5.8 to show the efficacy of the developed controllers. Conclusions are presented in Section 5.9.

5.2 Objectives of the Chapter

- To develop leader-follower flocking control law for a group of AUVs based on mathematical potential functions considering both without and with communication constraints.

5.3 Problem Formulation

The objective of this chapter is to design a distributed control law for each AUV such that the multi-AUV system steers to maintain connectivity during flocking starting from any initial connected topological configuration.

The positions of AUVs asymptotically converge to that of the desired trajectories

$$\lim_{t \to \infty} \eta_i \to \eta_d \tag{5.1}$$

where $\eta_d = [x_d, y_d, z_d, \varphi_d, \theta_d, \psi_d]^T$ is the position of the desired trajectory.

To maintain a certain distance between neighbours as well as to avoid collision with its flock-mates during manoeuvre. i.e.

$$\|\eta_i - \eta_j\| = d_{ij},\ \forall t \neq 0 \text{ and } d_{ij} \simeq d \tag{5.2}$$

$\|\bullet\|$ is the Euclidian norm, d_{ij} is the distance between i^{th} and its neighbour j^{th} AUVs. d is the desired safety distance. The distance between i^{th} and j^{th} AUV is always greater than that of a minimum distance so that there is no chance of collision between flock-mates. Similarly the distance between i^{th} and j^{th} AUV is always less than a maximum distance, hence the flock-mates will never diverse up to infinite and stay within the group. During flocking the distance among AUVs may be presented as shown in Figure 5.2.

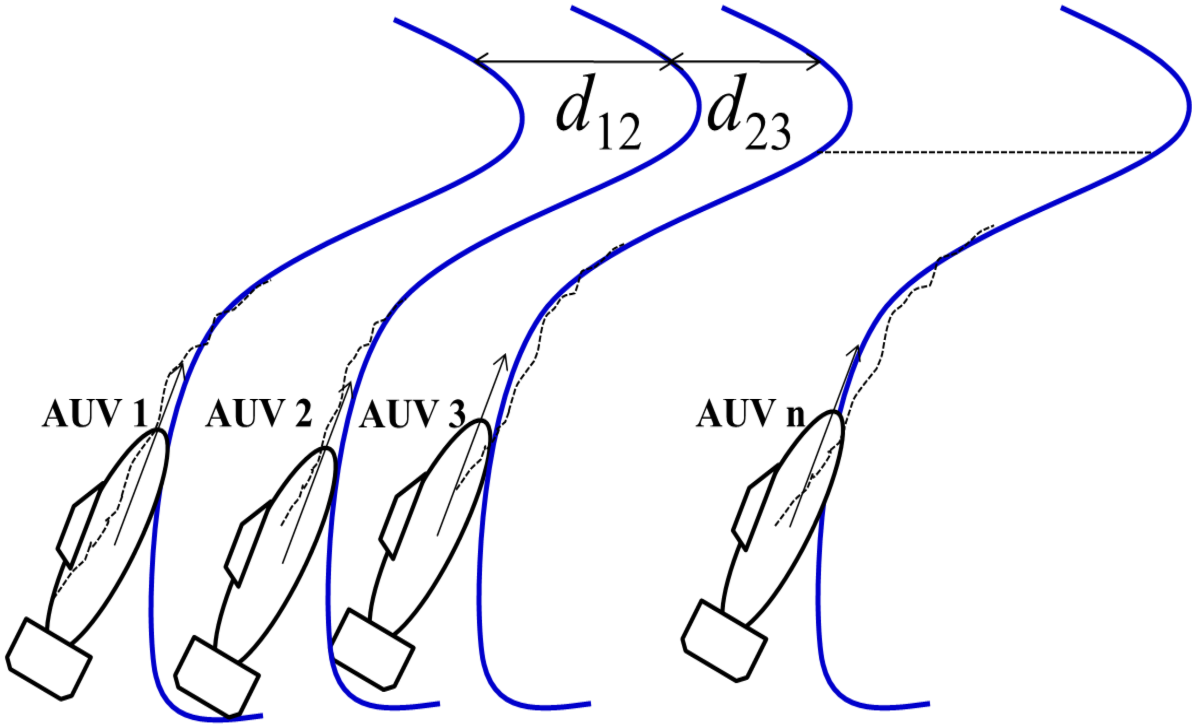


Figure 5.2 AUVs flocking along the desired trajectory keeping a minimum distance between them to avoid collision

To avoid obstacles, the distance between AUV and obstacle should greater than that of a safety distance i.e.

$$\lim_{t\to\infty}\|\eta_i-\eta_m\|>0 \tag{5.3}$$

$\eta_m=[x_m,y_m,z_m,\varphi_m,\theta_m,\psi_m]^T$ is the position vector of m^{th} obstacle, $\left(m\in\{1,2,3,...,n_{obs}\}\right)$, n_{obs} is the number of obstacles.

5.4 Kinematic and Dynamic of AUV

The motion of i^{th} AUV in six degrees of freedom (DOF) can be achieved by referring equation (3.3) through (3.8) and presented in a different manner as per necessary as in (4.2).

The dynamical and kinematic equations of motion can be presented by the following equation

$$\begin{aligned} &\dot{\eta}_i = J_i\left(\eta_i\right)\nu_i \\ &M_i\dot{\nu}_i + C_i(\nu_i)\nu_i + D_i(\nu_i)\nu_i + g_i(\eta_i) = \tau_i \end{aligned} \tag{5.4}$$

where the parameters $M_i(\eta_i)$, $C_i(\eta_i,\dot{\eta}_i)$, $D_i(\eta_i,\dot{\eta}_i)$, $g_i(\eta_i)$, $J_i(\eta_i)$ are highly nonlinear and coupled parameters which are mentioned and explained in Section 3.4.

5.5 Graph theory and Consensus Algorithm

5.5.1 Graph Theory

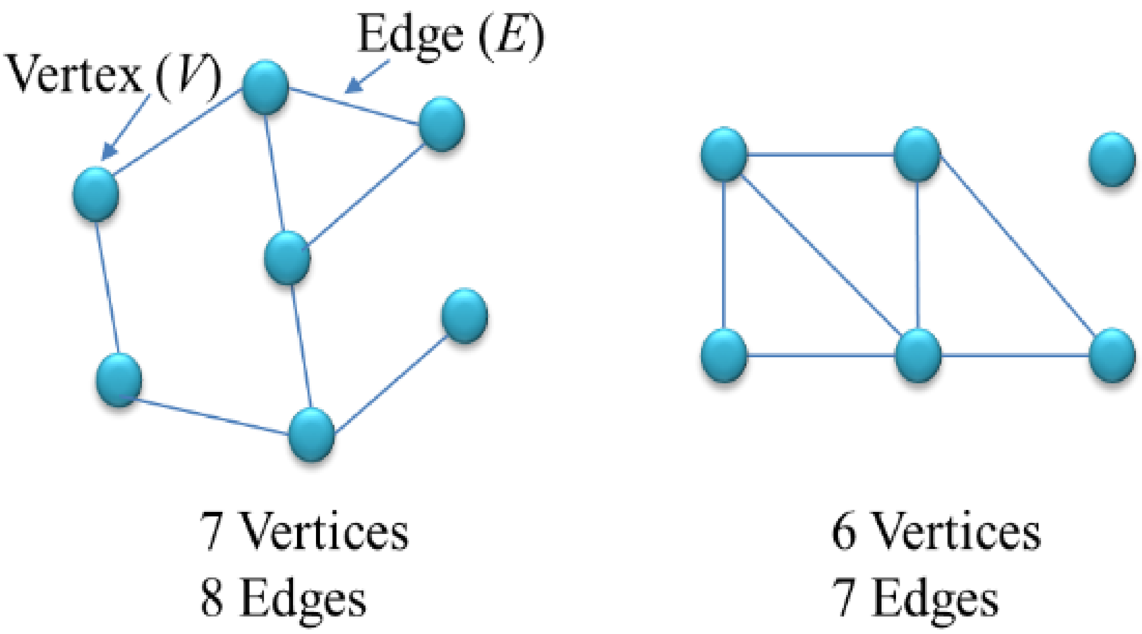


Figure 5.3: Graph showing vertices and edges

To develop flocking control law for multiple AUVs, graph theory based basic definitions are briefly presented here. As the information exchange between AUVs is symmetrical, only the undirected graph is explained. An undirected graph G consists of a pair of sets $(\mathcal{V}, E)$ where $\mathcal{V}$ is the finite nonempty set of elements called vertices, $E\left(E \in \mathcal{V} \times \mathcal{V}\right)$ is the set of unordered pair of distinct vertices called edge (Figure 5.3) which denotes the communication relation between AUVs [129]. The vertex set $\mathcal{V}$ and the edge set E are expressed as $\mathcal{V}(G)$

and $E(G)$ respectively. If $i, j \in \mathcal{V}$ and $(i, j) \in E$, then *i* and *j* are said to be adjacent or neighbours and this can be denoted by *i~j*. The number of neighbours of a vertex is its valence. A path of length S from vertex *i* to *j* is a sequence of $S+1$ vertices starting from the vertex *i* and ending at *j*. These vertices are arranged in such a manner that the consecutive vertices are adjacent. The set $E(G)$ may be defined as

$$E = E(G) = \left\{(i, j) \mid \left\|\eta_i - \eta_j\right\| < r_c, i, j \in \mathcal{V}, i \neq j\right\} \tag{5.5}$$

where r_c is the maximum allowed distance between the two AUVs beyond which they will disconnect from group.

A graph G is said to be connected or weakly connected if in between any two vertices *i* and *j*, there exists a path from *i* to *j*. Otherwise, if it is a directed path and there exists a path from each vertex to every other vertex, then it is a strongly connected graph.

The adjacency matrix represents, which vertex is adjacent to which other vertices. The adjacent matrix $A(G) = \left[a_{ij}\right]$ of an undirected graph G is a symmetric matrix with rows and columns indexed by vertices. The elements of the matrix

$$a_{ij} = \begin{cases} 1 \text{ if } i^{\text{th}} \text{ vertex and } j^{\text{th}} \text{ vertex are neighbors} \\ 0 \qquad\qquad\qquad\qquad\qquad\qquad \text{otherwise} \end{cases} \tag{5.6}$$

From (5.6) it can be observed that the set of edges connected with i^{th} AUV by E_i, then the communication neighbour set of i^{th} AUV is given by

$$C_i = C_i(G) = \{ j \in \mathcal{V} \mid (i, j) \in E_i \}. \tag{5.7}$$

The cardinality of C_i denotes the number of edges that can be connected to the i^{th} AUV. The degree matrix $D(G)$ of a graph G is the diagonal matrix $\{\text{dig}(ii)\}$ which has the information about each vertex. The undirected graph of a symmetric matrix is a graph whose adjacent matrix can be constructed as nonzero entries of the matrix with 1.

The Laplacian matrix $L(G)$ may be defined as

$$L = L(G) = D(G) - A(G) \tag{5.8}$$

The Laplacian matrix possesses many topological properties and is a semi-definite matrix. The algebraic multiplicity of its eigenvalue is equal to the number of connected components in the graph. The n-dimensional eigenvector associated with the zero eigenvalue is the vector of ones. Hence $L(G)$ is always positive semi-definite and symmetric for undirected graph G and satisfies the quadratic form $b^T Lb$ given by

$$b^T Lb = \frac{1}{2}\sum_{i=1}^{n}\sum_{j\in C_i} a_{ij}\left(b_j - b_i\right)^2 \tag{5.9}$$

where $b = (b_1, b_2, \cdots, b_n)$ for connected undirected graph G, the Laplacian has a single eigen value and the corresponding eigenvector has the dimension of [1, 1, ...1]^T.

5.5.2 Consensus Theorem

Consider there are n AUVs in a group. Out of these n_l numbers of AUVs are considered as leaders and n_f number of AUVs as followers. Hence $n = n_l + n_f$. Instead of followers following the leader AUV directly, the states of all vehicles in group converge towards a virtual point called the flocking centre. Each AUV sends its present state information to the neighbour AUVs so that, this information is the local information for every AUV in its locality. The idea behind the flocking algorithm as follows. All AUVs should remain within the flocking group. This is possible if each AUV is connected with its neighbour AUV through the flocking centre. The position of flocking centre approximates the average of the position of all AUVs in the group. All AUVs acquire this information about the flocking centre via communication topology among neighbour AUVs. This helps all AUVs to remain cohesive with the flockmates within the group. A consensus algorithm is presented here to keep neighbour AUVs connected with the group.

By following [73], [211], [230], [242], [244], [245] and (5.9), the appropriate consensus algorithms developed to estimated position of flocking centre η_i^c of i^{th} AUV as

$$\dot{\eta}_i^c = \sum_{j=1}^{n_{ni}}\left(\eta_i^c - \eta_i\right) + \left(\eta_j^c - \eta_i^c\right) \tag{5.10}$$

where η_j^c is the estimated position of flocking centre of j^{th} AUV. This can be presented in vector form as

$$\dot{\eta}^c = \eta - (L+1)\eta^c \tag{5.11}$$

where $\eta^c = \left[\eta_1^c, \eta_2^c, ..., \eta_n^c\right]^T$ is a matrix formed by the vectors of flocking centres of the individual AUVs, $\eta = \left[\eta_1, \eta_2, ..., \eta_n\right]^T$ is a matrix formed by the vectors of the positions of the individual AUVs.

The whole flocking group is to follow the desired trajectory i.e. the average centre of the group will follow the desired trajectory. The average position of coordinates (η_c) of all AUVs is defined as

$$\eta_c = \frac{1}{n}\sum_{i=1}^{n}\eta_i \ . \tag{5.12}$$

To follow the desired trajectory by AUVs, η^c should asymptotically converge to η_c and $\eta_i^c \to \eta_i$.

5.6 Potential Function based Control of AUVs

For path following control of multiple AUVs in flocking, a simple and very powerful method, i.e. the artificial potential function method is employed. In this method, an individual AUV is considered as a single isolated particle emerging in a potential well. The potential field is created from two sources such as the potential field between AUVs themselves as well as AUV and desired trajectory points. The collective potential function is a nonnegative function $U(\eta)$. The distance r_{ij} between i^{th} and j^{th} AUV is represented as $r_{ij} = \left\|\eta_i - \eta_j\right\|$. Hence, one of the properties of potential function is the algebraic distance r_{ij} is closely related to $U(\eta)$. The collective potential function is a function of distance. For flocking control of multiple AUVs, the following forces are necessary for keeping the multi-AUV system connected and remaining in the group. To avoid collision among AUVs a repulsive potential function (RPF) is introduced within the group.

5.6.1 Attractive Potential Function (APF)

To avoid group splitting, there should exist an APF between the members. This is an inter AUV distance dependent function. This value should be zero when $\eta_i = \eta_j$ and possess maximum value when $\left\|\eta_i - \eta_j\right\| \geq d$. d is the safety distance. In this case, all the AUVs of the group are oriented towards a common flocking centre. Hence, the potential function is a function of the position of AUVs as well as their positions of flocking centres. It may be represented for the i^{th} AUV as $U_{i,att}^{c}\left(\eta_i, \eta_i^c\right)$.

Consider the APF for the i^{th} AUV as

$$U_{i,att}^{c}\left(\eta_i, \eta_i^c\right) = \frac{1}{2} k_{att}^{c} {r_{ic}}^2 \tag{5.13}$$

where $r_{ic} = \left\|\eta_i - \eta_i^c\right\|$ is the Euclidean distance between position of i^{th} AUV and the flocking centre corresponding to that AUV. Let k_{att}^{c} is a positive scaling factor. The gradient of this function can be represented as

$$\nabla U_{i,att}^{c}\left(\eta_i, \eta_i^c\right) = k_{att}^{c}\left(\eta_i - \eta_i^c\right) \tag{5.14}$$

The force of attraction between the position of the i^{th} AUV and position of corresponding flocking centre is the negative gradient of attractive potential between them and can be written as [211], [240], [249],

$$F_{i,att}^{c} = -\nabla U_{i,att}^{c}\left(\eta_i, \eta_i^c\right) = -k_{att}^{c}\left(\eta_i - \eta_i^c\right) \tag{5.15}$$

Similarly an APF between i^{th} AUV and desired trajectory is taken as

$$U_{i,att}^{d}\left(\eta_i, \eta_d\right) = \frac{1}{2} k_{att}^{d} {r_{id}}^2 \tag{5.16}$$

where $r_{id} = \left\|\eta_i - \eta_d\right\|$ is the Euclidean distance between position of i^{th} AUV and the desired trajectory. k_{att}^{d} is a positive scaling factor.

The potential function $F_{i,att}^{d}$ between the position η_i of i^{th} AUV and the coordinated position of the desired trajectory η_d is given by

$$F_{i,att}^{d} = -\nabla U_{i,att}^{d}\left(\eta_i,\eta_d\right) = -k_{att}^{d}\left(\eta_i - \eta_d\right) \tag{5.17}$$

5.6.2 Repulsive Potential Function (RPF)

This function has the maximum value when there is a chance of collision between AUVs i.e. when $\eta_i = \eta_j$ and asymptotically converges to zero when $\left\|\eta_i - \eta_j\right\| \geq d$. Hence AUVs would able to maintain a minimum certain safety distance among them. The RPF between i^{th} and j^{th} AUV can be chosen as $U_{rep}^{a}(\eta_i,\eta_j)$. This function for the i^{th} AUV due to j^{th} AUV in the group is $\sum_{j=1}^{n_{ni}} U_{i,rep}^{a}\left(\eta_i,\eta_j\right) i \neq j$.

If any AUV (either leader or follower) is at a larger distance than the safety distance from another AUV, the first AUV will unable to repel the second one. However, the repulsive potential increases gradually when the AUVs approach near to each other and diminutions gradually when they diverse from each other. At a certain maximum distance the repulsive potential becomes zero. The negative gradient of the RPF for i^{th} AUV $F_{i,rep}^{a}$ is given by

$$F_{i,rep}^{a} = -\sum_{j=1}^{n_{ni}} \nabla U_{i,rep}^{a}\left(\eta_i,\eta_j\right) \;\; i \neq j \tag{5.18}$$

In an obstacle rich environment, there should a repulsive potential between AUV and obstacles. In the similar way the negative gradient of potential function due to i^{th} AUV $F_{i,rep}^{o}$ can be found as

$$F_{i,rep}^{o} = -\sum_{j=1}^{n_{obs}} \nabla U_{rep}^{o}\left(\eta_i,\eta_j^{o}\right) \tag{5.19}$$

where n_{obs} is the number of obstacles, η_j^{o} is the position of j^{th} obstacle, $\eta_j^{o} = \left[x_j^{o}, y_j^{o}, z_j^{o}, \phi_j^{o}, \theta_j^{o}, \psi_j^{o}\right]^T$, $U_{rep}^{o}\left(\eta_i,\eta_j^{o}\right)$ is the repulsive potential between the i^{th} AUV and j^{th} obstacle.

The total potential function for i^{th} AUV can be obtained as the summation of the attractive as well as repulsive potentials. The negative gradient of the potential function for i^{th} AUV may be represented as

$$F_i = F_{i,att}^c + F_{i,att}^d + F_{i,rep}^a + F_{i,rep}^o \tag{5.20}$$

Substituting (5.15), (5.17), (5.18) and (5.19) in (5.20) one obtains

$$F_i = -k_{att}^c\left(\eta_i - \eta_i^c\right) - k_{att}^d\left(\eta_i - \eta_d\right) - \sum_{j=1}^{n_{ni}} \nabla U_{i,rep}^a\left(\eta_i, \eta_j\right) - \sum_{j=1}^{n_{obs}} \nabla U_{rep}^o\left(\eta_i, \eta_j^o\right) \tag{5.21}$$

The control law for i^{th} leader AUV can be represented as

$$\dot{\eta}_i^l = F_i^l + \dot{\eta}_d\,. \tag{5.22}$$

The states are considered only for the i^{th} leader AUVs in (5.22). Here

$$F_i^l = -k_{att}^c\left(\eta_i^l - \eta_i^{lc}\right) - k_{att}^d\left(\eta_i^l - \eta_d\right) - \sum_{j=1}^{n_{ni}} \nabla U_{i,rep}^a\left(\eta_i, \eta_j\right) - \sum_{j=1}^{n_{obs}} \nabla U_{rep}^o\left(\eta_i^l, \eta_j^o\right) \tag{5.23}$$

where η_i^{lc} is the flocking centre for the i^{th} leader AUV.

Follower AUVs do not have any global information about the desired trajectory. Hence, the controller for the i^{th} follower AUV may be represented as

$$\dot{\eta}_i^f = F_i^f \tag{5.24}$$

with

$$F_i^f = -k_{att}^c\left(\eta_i^f - \eta_i^{fc}\right) - k_{att}^d\left(\eta_i^f - \eta_d\right) - \sum_{j=1}^{n_{ni}} \nabla U_{i,rep}^a\left(\eta_i, \eta_j\right) - \sum_{j=1}^{n_{obs}} \nabla U_{rep}^o\left(\eta_i^f, \eta_j^o\right) \tag{5.25}$$

where η_i^{fc} denotes the flocking centre of the i^{th} follower AUV. The stability of the closed loop system is given in this Section later.

The term which is common to both (5.23) and (5.25) i.e. $\sum_{j=1}^{n_{ni}} \nabla U_{i,rep}^{a}\left(\eta_i, \eta_j\right)$ and $\sum_{j=1}^{n_{obs}} \nabla U_{rep}^{o}\left(\eta_i, \eta_j^{o}\right)$ denotes the RPF for the i^{th} leader or follower AUV. Here the RPF between AUVs to avoid collision among them is developed in this chapter by using both mathematical and fuzzy variable methods.

The RPF can be derived in different ways. We consider the following mathematical RPF (MRPF) for i^{th} AUV as [200],

$$U_{i,rep}^{a} = \frac{1}{\left(\frac{\left\|r_{ij}\right\|}{d}\right)^2} + \log\left(\frac{\left\|r_{ij}\right\|}{d}\right)^2 \tag{5.26}$$

The negative gradient for x-coordinate of MRPF described in (5.26) may be presented by

$$F_{i,rep}^{a,x} = -\nabla U_{i,rep}^{a} = -\frac{2\left(x_i - x_j\right)\left(r_{ij}^2 - d^2\right)}{r_{ij}^4} \tag{5.27}$$

where x_i and x_j are the x component of the i^{th} and j^{th} AUVs respectively. r_{ij} is the distance between i^{th} and j^{th} AUV. d is the desired safety distance taken between AUVs.

Similarly the MRPF and its negative gradient between the i^{th} AUV and the obstacle may be given as

$$U_{i,rep}^{o,x} = \frac{1}{\left(\frac{\left\|r_{ij_{obs}}\right\|}{d}\right)^2} + \log\left(\frac{\left\|r_{ij_{obs}}\right\|}{d}\right)^2$$

and

$$F_{i,rep}^{o,x} = -\nabla U_{rep}^{o} = -\frac{2\left(x_i - x_{j_{obs}}\right)\left(r_{ij_{obs}}^2 - d^2\right)}{r_{ij_{obs}}^4} \tag{5.28}$$

where $x_{j_{obs}}$ is the x component of the j^{th} obstacle. $r_{ij_{obs}}$ is the distance between i^{th} AUV and j^{th} obstacle.

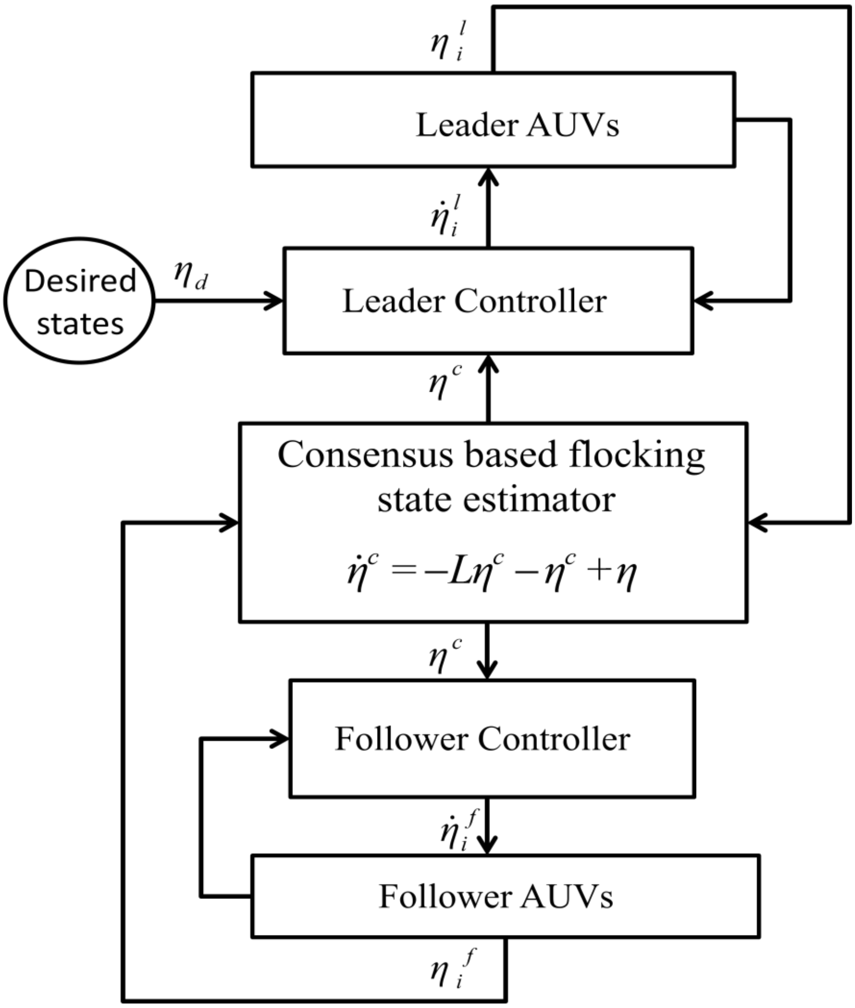


Figure 5.4: Control structure for flocking of AUVs

5.7 Development of Control law with Communication Constraints

In this section, control law for flocking of a group of AUVs in the presence of communication constraints and noise is developed. Keeping communication under water is not easy due to several reasons such as chances of multiple propagation, small available bandwidth, uncertainty or time variation in channel of propagation, strong attenuation of signal in travelling medium, etc.. Generally in AUV communication, acoustic signals are used instead of electromagnetic waves like in terrestrial communication. One demerit of acoustic communication is the transmission of low data rate compared to terrestrial communication.

The problem of updating of estimated data in communication issue may be solved by using continuous-discrete Extended Kalman Filter (EKF) algorithm. The states of one AUV propagate in water in the discrete form. The rest of the AUVs in group may not receive these data in the exact form because of data loss in propagation medium. So it is necessary to estimate the states to compensate the losses.

For estimating the position of an AUV, let us assume an approximate model of the AUV as follows [267].

The system model is presented by

$$\dot{X} = f(X(t),t) + W(t) \tag{5.29}$$

The measurement model is

$$Z_k = h_k(X(t_k)) + V_k \tag{5.30}$$

where $X(t) = \eta_i(t)$, $W(t)$ and V_k are uncorrelated Gaussian white noise with zero mean.

The estimation state propagation model is

$$\dot{\hat{X}} = f(\hat{X}(t),t) \tag{5.31}$$

The error covariance propagation model is

$$\dot{P}(t) = F(\hat{X}(t),t)P(t) + P(t)F^T(\hat{X}(t),t) + Q(t) \tag{5.32}$$

where *P(t)* is the error covariance matrix. $F(\hat{X}(t),t) = \left.\dfrac{\partial f(X(t),t)}{\partial X(t)}\right|_{X(t)=\hat{X}_k(-)}$

The old data are updated with the new data from the continuous-discrete EKF. This updating algorithm is given as;

State estimation model:

$$\hat{X}_k(+) = \hat{X}_k(-) + K_k\left[Z_k - h_k(\hat{X}_k(-))\right] \tag{5.33}$$

Error covariance propagation:

$$P_k(+) = [I - K_k H_k(\hat{X}_k(-)]P_k(-) \tag{5.34}$$

The gain matrix:

$$K_k = P_k(-)H_k^{\;T}(\hat{X}_k(-))\left[H_k(\hat{X}_k(-))P_k(-)H_k(\hat{X}_k(-) + R_k\right] \tag{5.35}$$

with

$$H_k(\hat{X}_k(-)) = \left.\frac{\partial h_k(X(t_k))}{\partial X(t_k)}\right|_{X(t_k)=\hat{X}_k(-)}$$

As in this state estimation process $X(t) = \eta_i(t)$, so $\hat{X}(t) = \hat{\eta}_i(t)$.

In the flocking method, i^{th} AUV estimates the position of the neighbor flockmates i.e. of j^{th} AUV. So for i^{th} AUV $\eta_j(t) \rightarrow \hat{\eta}_j(t)$.

Using this estimated state the control law can be modified as follows.

The negative gradient of the repulsive potential function field is given by from (5.18),

$$\hat{F}^{a}_{i,rep} = -\sum_{i=1}^{n_{ni}}\sum_{j=1}^{n_{ni}} \nabla U^{a}_{rep}\left(\eta_i, \hat{\eta}_j\right) \quad i \neq j \tag{5.36}$$

Here the positions of the obstacles are considered to be known to all AUVs. So it is not necessary to estimate the states of the obstacles.

The total potential function of the whole system can be obtained as;

$$\hat{F} = -k^{c}_{att}\left(\eta_i - \eta^{c}_i\right) - k^{d}_{att}\left(\eta_i - \eta_d\right) - \sum_{i=1}^{n_{ni}}\sum_{j=1}^{n_{ni}} \nabla U^{a}_{rep}\left(\eta_i, \hat{\eta}_j\right) - \sum_{i=1}^{n_{ni}}\sum_{j=1}^{n_{obs}} \nabla U^{o}_{rep}\left(\eta_i, \eta^{o}_j\right) \tag{5.37}$$

where

$$\hat{U}^{a}_{rep} = \frac{1}{\left(\dfrac{\left\|\hat{r}_{ij}\right\|}{d}\right)^2} + log\left(\dfrac{\left\|\hat{r}_{ij}\right\|}{d}\right)^2 \tag{5.38}$$

where $\hat{r}_{ij} = \left\|\eta_i - \hat{\eta}_j\right\|$

The negative gradient for *x*-coordinate of potential function described in Section 5.6 is presented by

$$\hat{F}^{a}_{rep} = -\nabla U^{a}_{rep} = -\frac{2\left(x_i - \hat{x}_j\right)\left(\hat{r}_{ij}^{\,2} - d^2\right)}{\hat{r}_{ij}^{\,4}} \tag{5.39}$$

$\hat{x}_j$ is the estimated *x* component of j^{th} AUV.

$$\dot{\eta}^{l}_i = \hat{F}^{l} + \dot{\eta}_d \tag{5.40}$$

The states are considered only for leader vehicles in (5.40). Here

$$\hat{F}^{l} = -k^{c}_{att}\left(\eta^{l}_i - \eta^{lc}_i\right) - k^{d}_{att}\left(\eta^{l}_i - \eta_d\right) - \sum_{i=1}^{n_{ni}}\sum_{j=1}^{n_{ni}} \nabla \hat{U}^{a}_{rep}\left(\eta_i, \eta_j\right) - \sum_{i=1}^{n_{ni}}\sum_{j=1}^{n_{ni}} \nabla U^{o}_{rep}\left(\eta_i, \eta^{o}_j\right) \tag{5.41}$$

where η^{lc}_i the flocking centre for leader AUVs.

The controller for follower AUVs may be represented as

$$\dot{\eta}_i^f = \hat{F}^f \tag{5.42}$$

$$\hat{F}^f = -k_{att}^c\left(\eta_i^f - \eta_i^{fc}\right) - k_{att}^d\left(\eta_i^f - \eta_d\right) - \nabla\sum_{i=1}^{n_{ni}}\sum_{j=1}^{n_{ni}}\hat{U}_{rep}^a\left(\eta_i,\eta_j\right) - \nabla\sum_{i=1}^{n_{ni}}\sum_{j=1}^{n_{obs}}U_{rep}^o\left(\eta_i,\eta_j^o\right) \tag{5.43}$$

This controller (5.40) and (5.42) are used to enable the AUVs to flock along the desired path considering the communication constraints.

5.8 Results and Discussions

A simulation set up is made for studying efficacy of the proposed the leader-follower flocking algorithm as follows. Different cases are considered for flocking control of four AUVs. In one case all the four AUVs are considered as leader vehicles, i.e., each AUV has global knowledge of the desired path. In another case out of four AUVs in a group, only one AUV is considered as the leader vehicle and other AUVs are considered as followers. The simulation is carried out for the mathematical RPF methods. In all these cases the desired trajectory is defined as a circular path of radius 10m and in a space of height 10m which is given by

$$\begin{aligned} x_d &= 10\sin(0.01\text{t}) \\ y_d &= 10\cos(0.01\text{t})\,. \\ z_d &= 10,\ \psi_d = \frac{\pi}{3} \end{aligned} \tag{5.44}$$

The constants are taken as $k_{att}^c = k_{att}^d = 10,\ d = 2m$.

The pitch and yaw orientations are neglected here as these parameters are not much imperative and simulations are carried out considering four DOFs.

Two cases are considered for the simulation of multiple AUVs based on mathematical RPFs. In the first case, all the four AUVs are considered as leaders and in another case one AUV is taken as leader and other three AUVs are considered as followers.

5.8.1 Flocking of Four AUVs (All are Leaders) using Mathematical RPF in Circular Path

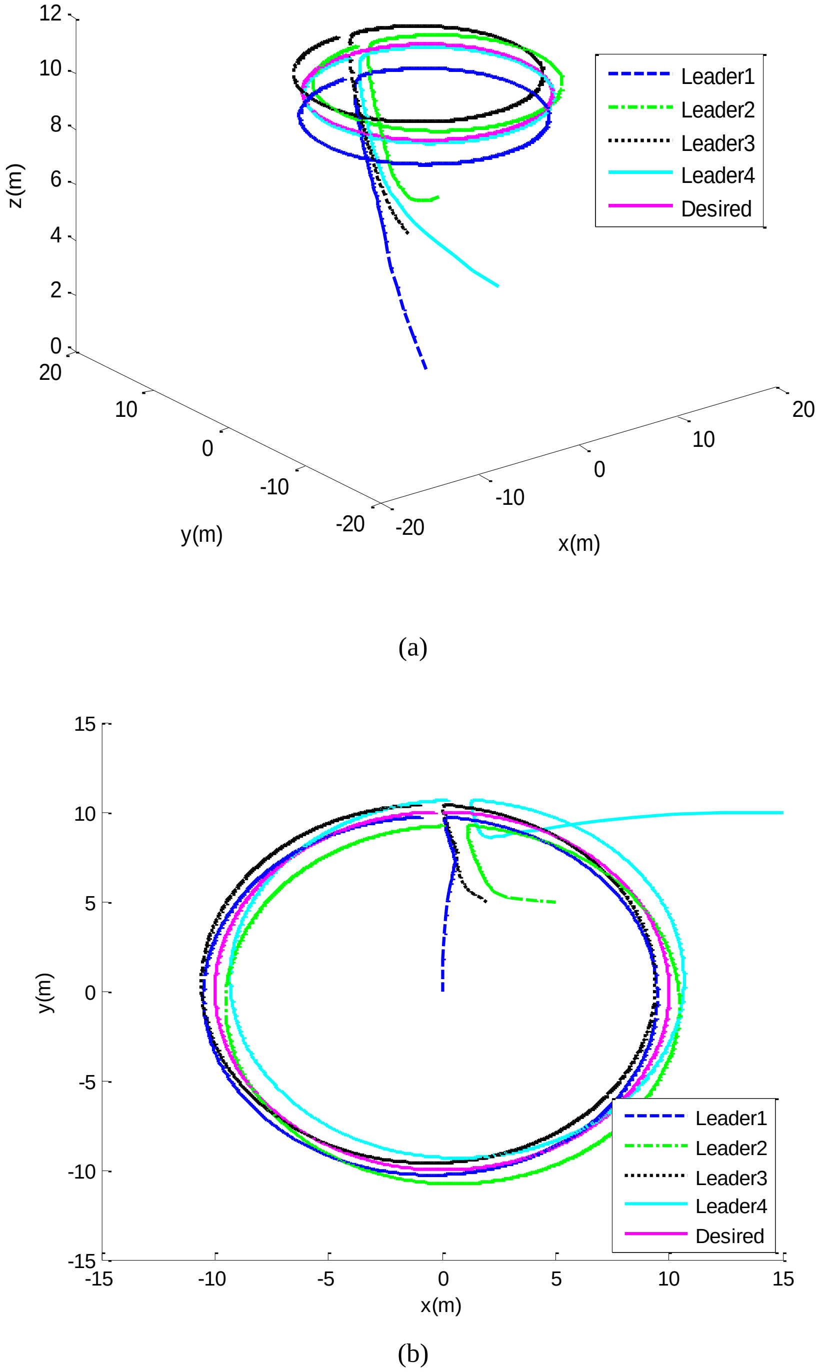
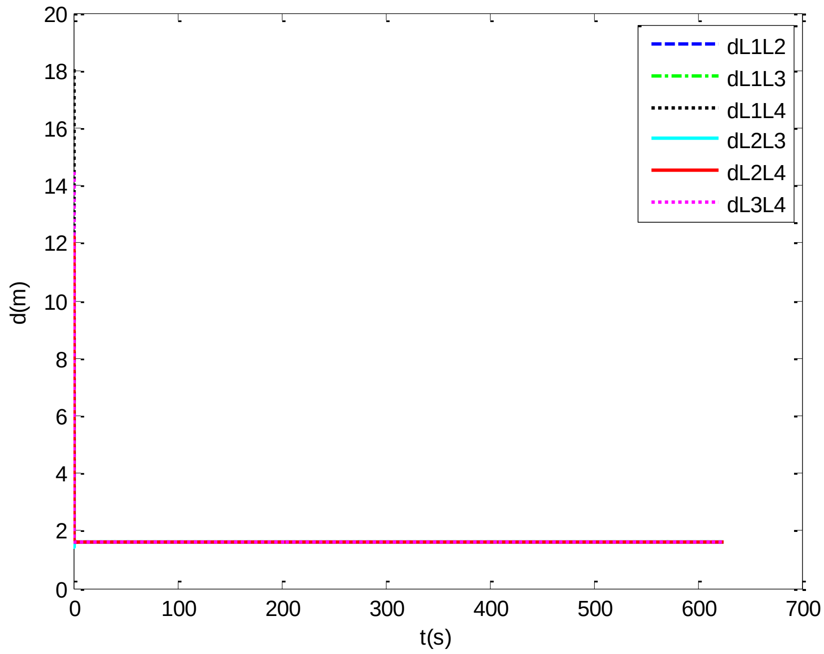
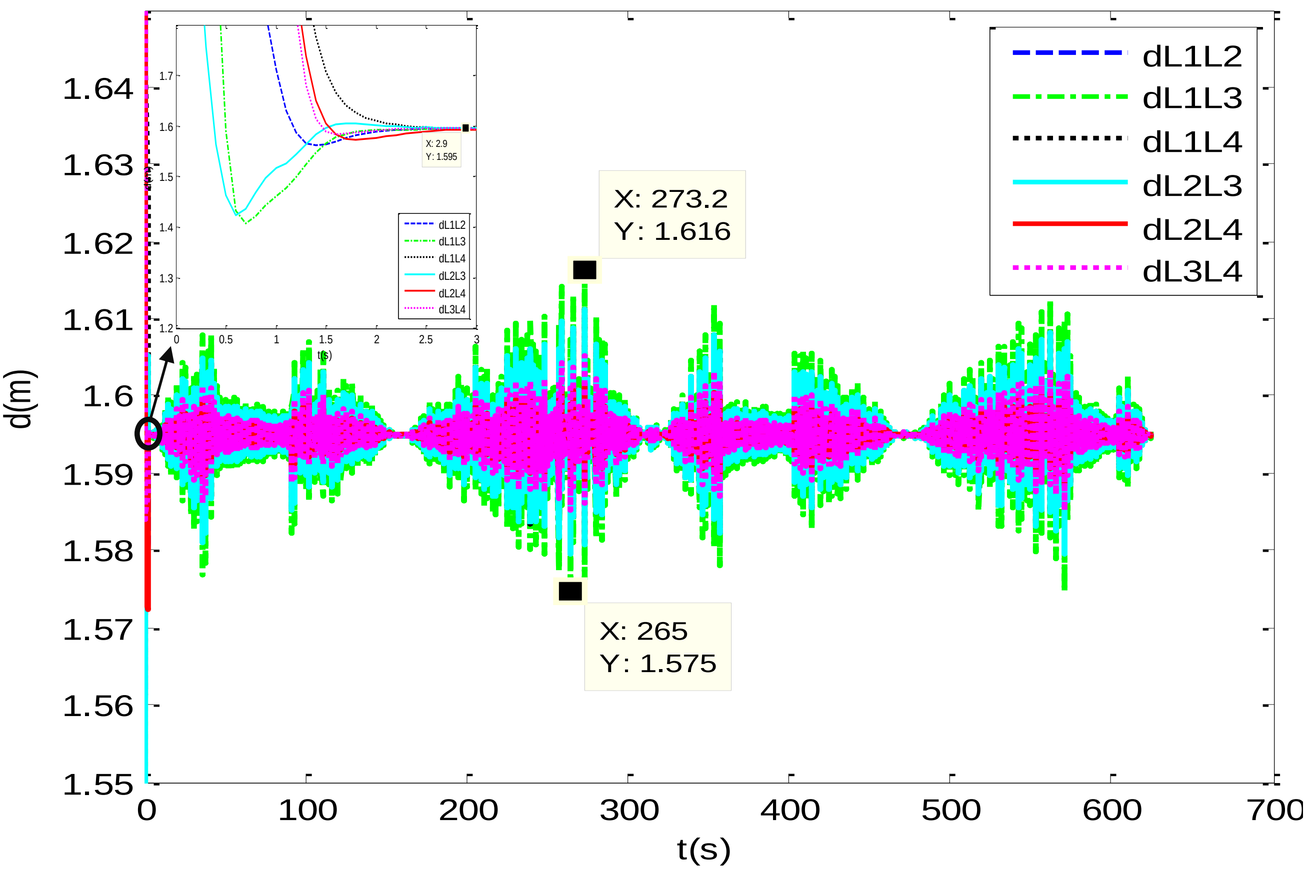


Figure 5.5: Flocking of four AUVs (all are leaders) is in circular path using RPF in (a) space (b) plane.



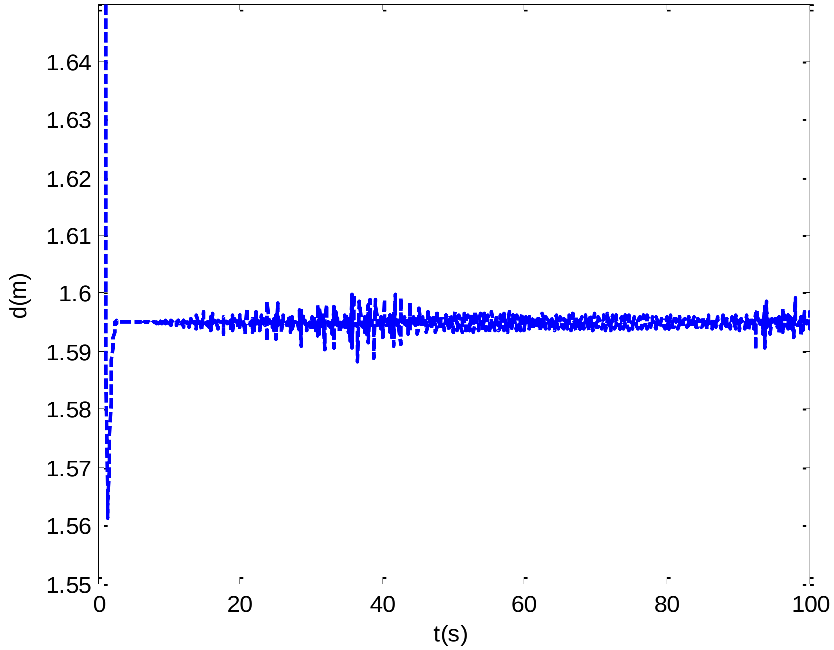
dL1L2
dL1L3
dL1L4
dL2L3
dL2L4
dL3L4
d(m)
t(s)

(a)



dL1L2
dL1L3
dL1L4
dL2L3
dL2L4
dL3L4
X: 273.2
Y: 1.616
X: 265
Y: 1.575
X: 2.9
Y: 1.595
d(m)
t(s)

(b)



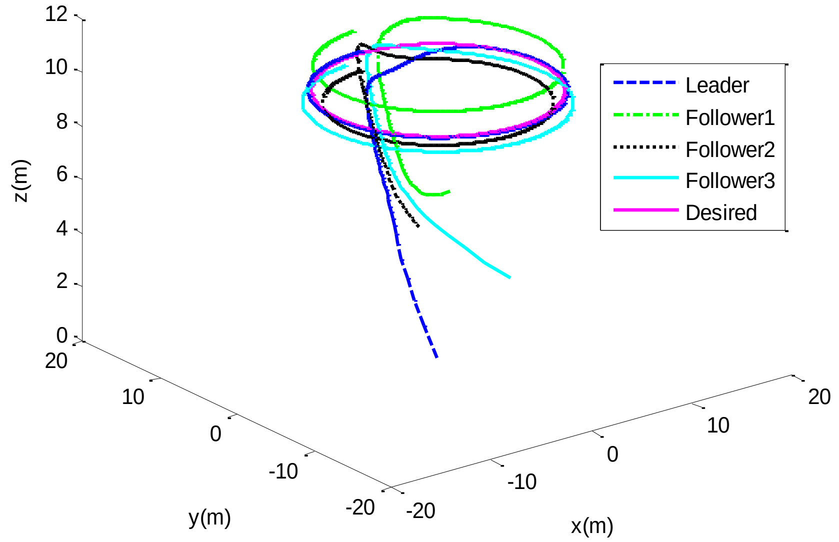
(c)

Figure 5.6: Distance among AUVs in (a) 650s (b) 650s (c) 100s (between one pair of leader AUVs)

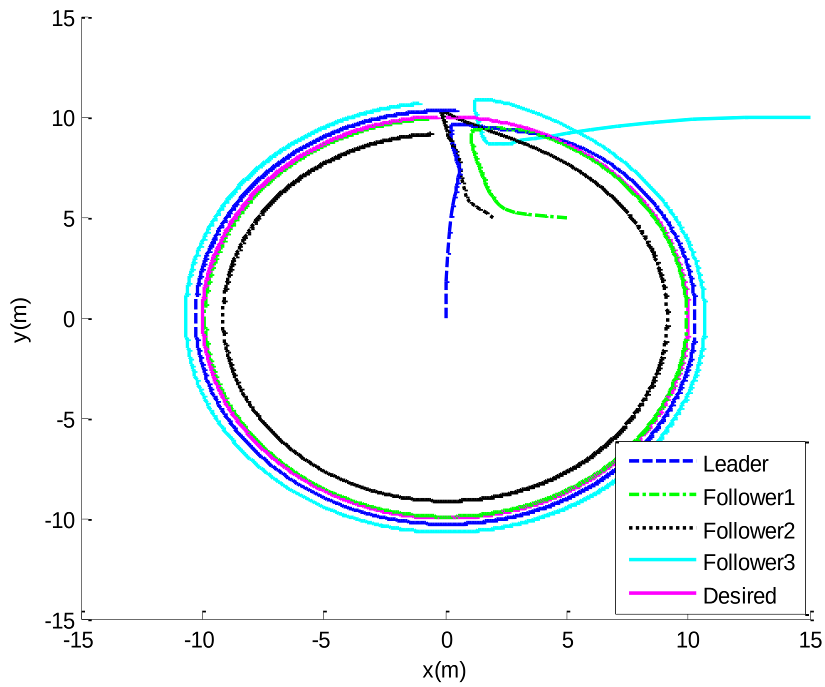
From Figure 5.5 it is observed that four AUVs are flocking along the desired path. When the mathematical separation function is used, AUVs separate from each other and avoid collision among them Figure 5.6(a), Figure 5.6(b), Figure 5.6(c). Figure 5.6(c) presents the separation distance between two leader AUVs i.e. leader AUV1 and leader AUV2. In these figures '▪' presents a point whose *x*-coordinate value is X and *y*-coordinate value is Y. From Figure 5.6, it is observed that the flocking approach time of the group of AUVs is 2.9 seconds.

5.8.2 Flocking of Four AUVs (One Leader and Three Followers) in Circular Path

Figure 5.7 presents flocking of AUVs along the desired track and. Figure 5.8 presents the distances maintained among the AUVs.

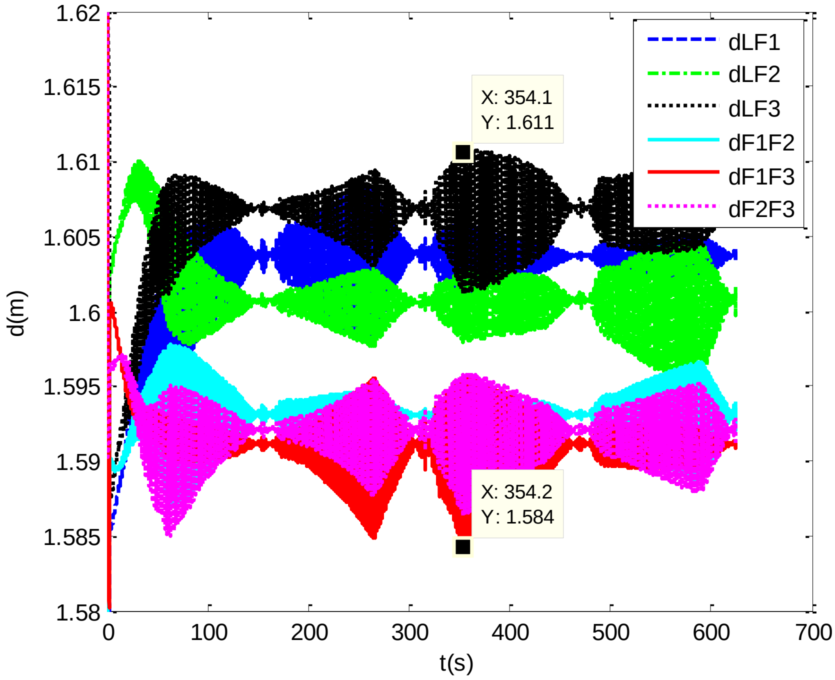


(a)

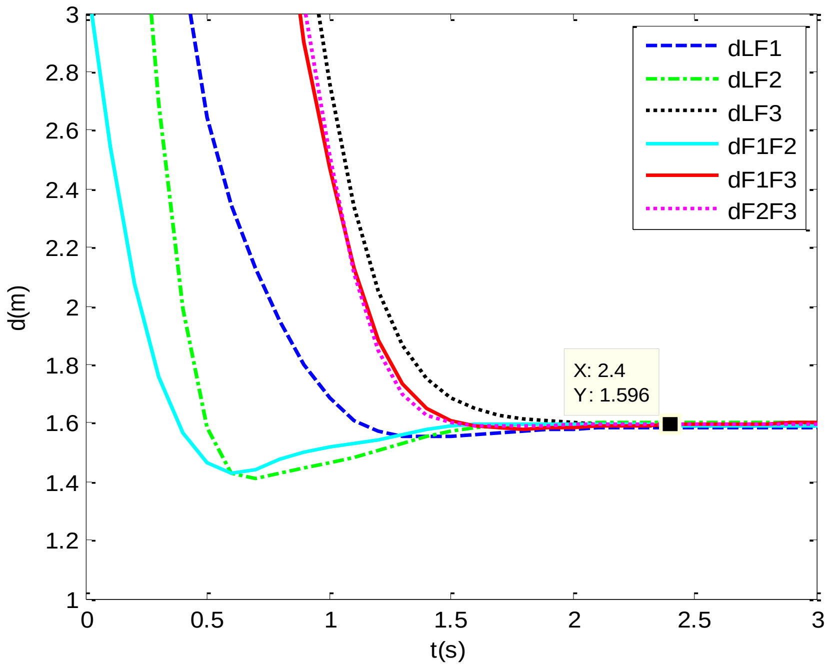


(b)

Figure 5.7: Flocking of AUVs (one leader and three followers) in circular path in (a) space (b) plane



(a)



(b)

Figure 5.8: Distance among four AUVs for case 2 in (a) 650s and (b) 3s

Figure 5.7 and Figure 5.8, it is observed that all the four AUVs are able to track the desired trajectory according to the leader-follower flocking law without any collision among them. The followers are able to follow the leader to track the desired trajectory.

5.8.3 Obstacle Avoidance of Four AUVs (One Leader and Three Followers) using Mathematical SPF

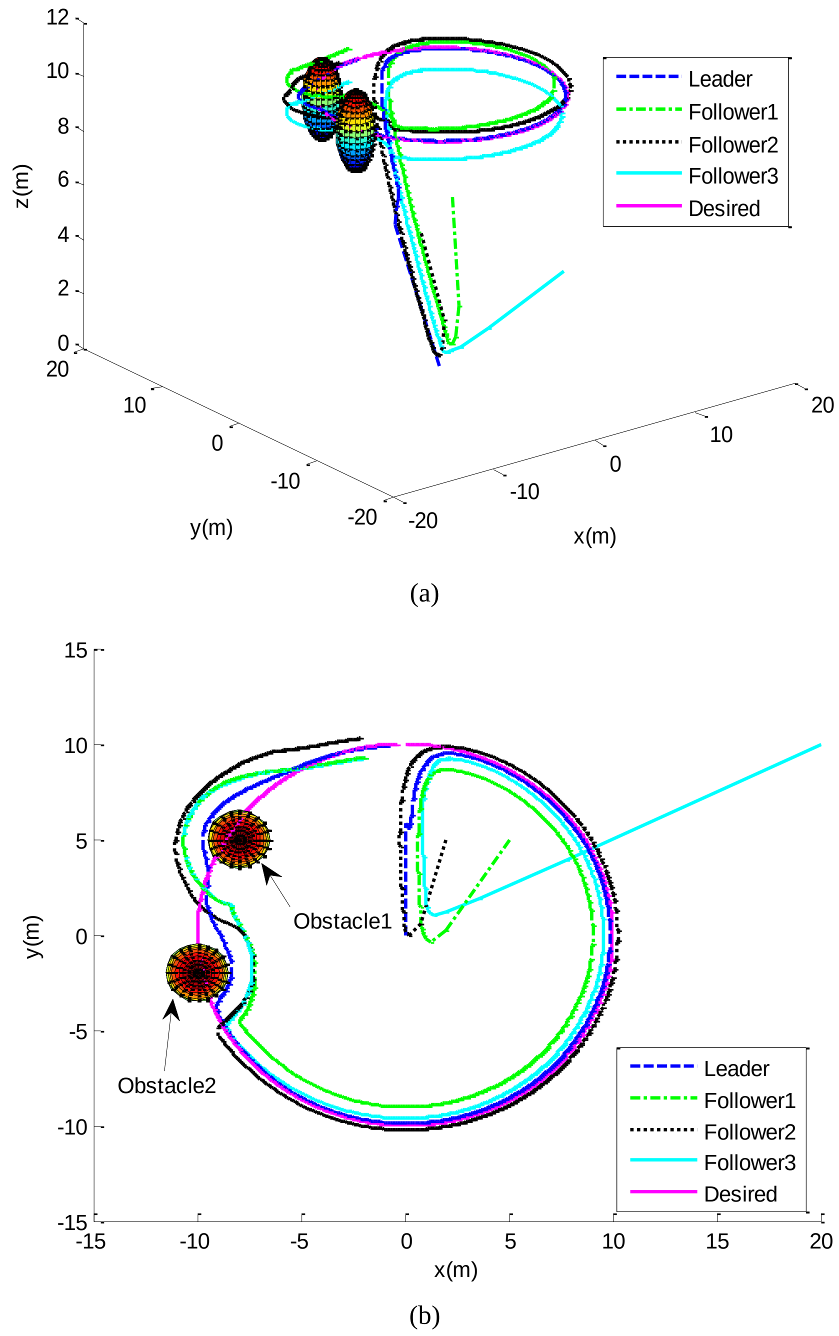


Figure 5.9: Flocking of four AUVs (one leader and three followers) travelling in a circular path and avoiding solid obstacles in obstacle-rich environment in (a) space (b) plane using mathematical potential functions

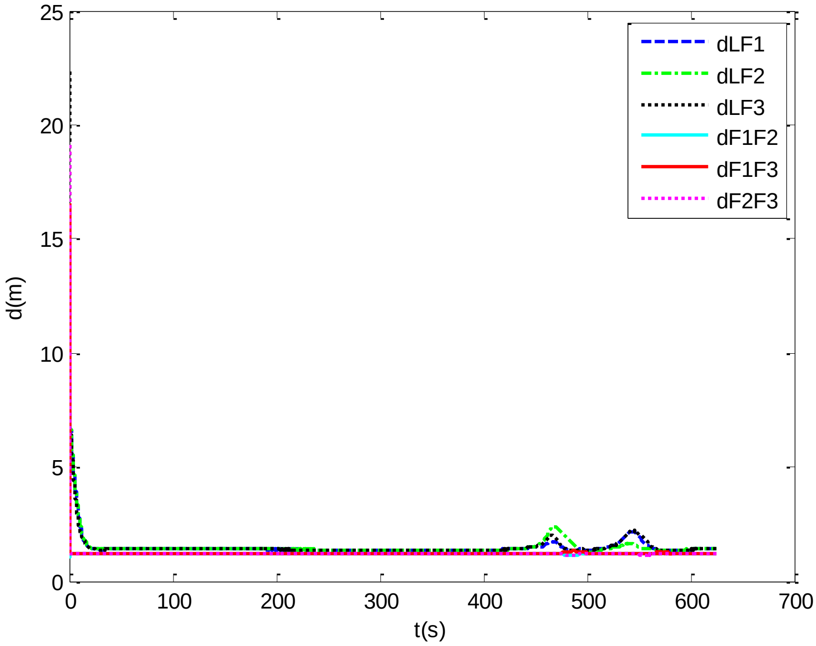
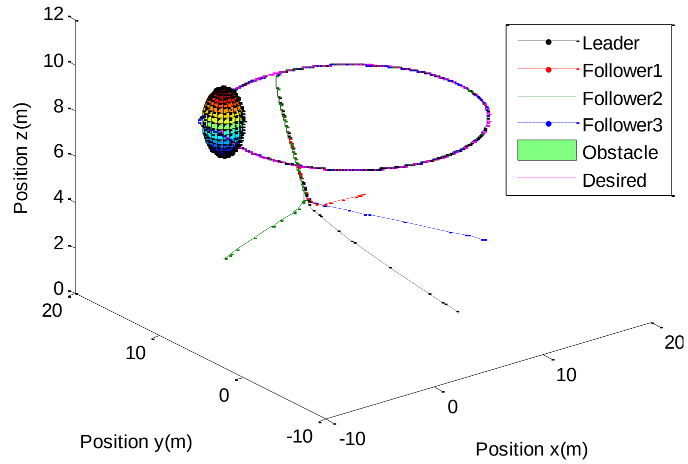


Figure 5.10: Distance among four AUVs (one leader and three followers) during Flocking in a circular path during obstacle avoidance using mathematical potential functions

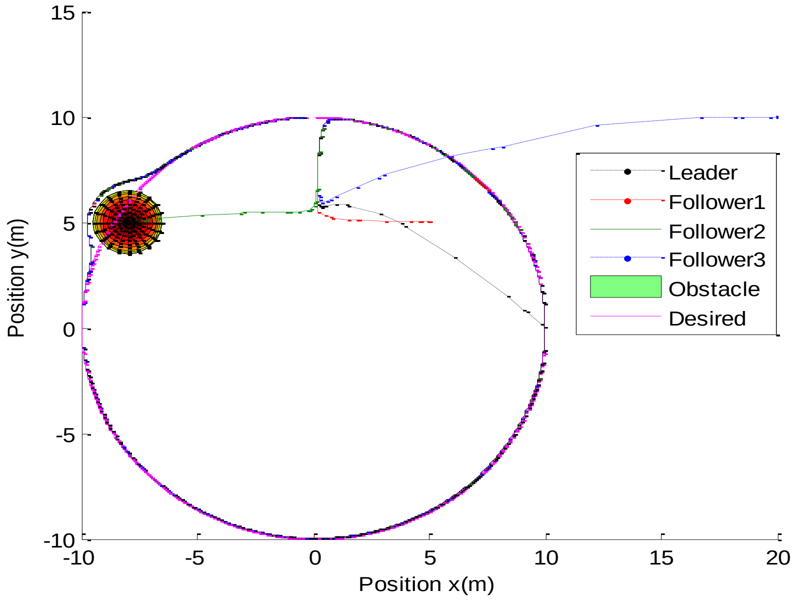
Figure 5.9 presents the flocking control and obstacle avoidance of group of AUVs having a single leader in an obstacle populated environment. From this figure, it is observed that the controller is able to flock the AUVs along the desired trajectory. The distance among the AUVs are presented in Figure 5.10.

5.8.4 Flocking results with communication constraints

Flocking of four AUVs (One leader & Three Followers) and Obstacle Avoidance



(a)



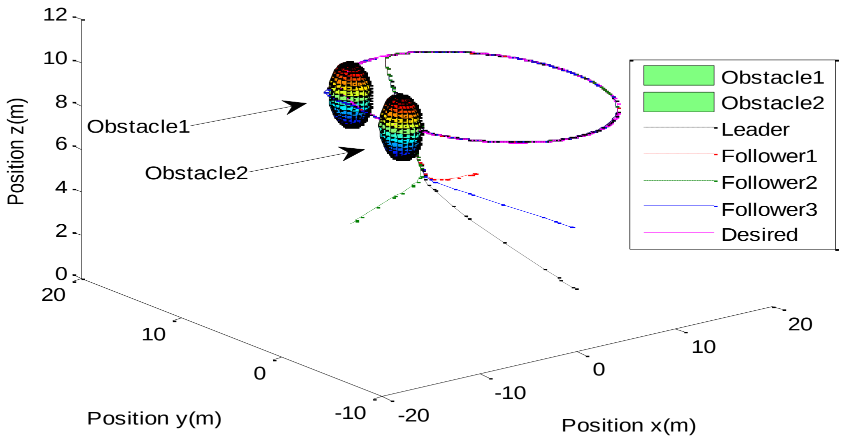
(b)

Figure 5.11: Flocking of four AUVs (one leader and three followers) travelling on the circular path and avoiding a solid obstacle in (a) space (b) plane considering communication constraints

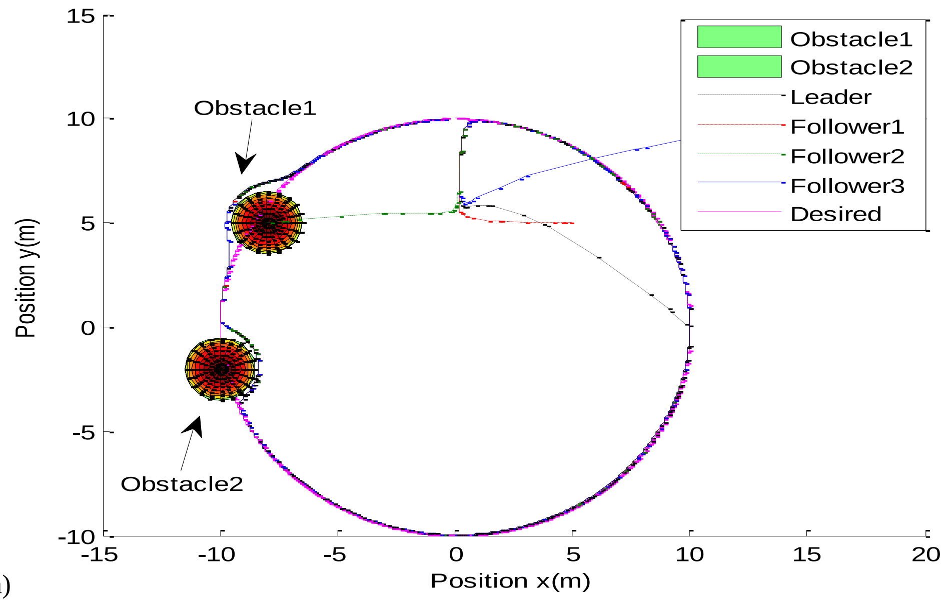
Figure 5.11 shows flocking of four AUVs (one leader and three followers) travelling on the circular path and avoiding a solid obstacle in (a) space (b) plane considering communication constraints

Results presented in Figure 5.11 show the flocking of four AUVs (one leader and three followers) travelling on the circular path in space. From these results it is clear that the AUVs are able to avoid the solid obstacle. The simulation is carried out considering communication constraints in the underwater environment.

Flocking of four AUVs (One leader & Three Followers) and in the obstacle rich environment



(a)



(a)

(b)

Figure 5.12: Flocking of four AUVs (one leader and three followers) travelling on the circular path in obstacle-rich environment in (a) space (b) plane considering communication constraints

Figure 5.12 depicts the results of flocking of four AUVs considering one leader and three followers travelling on the desired circular path in space and plane respectively. From these simulation results it is observed that the AUVs are able to avoid multiple obstacles is an obstacle rich environment considering communication constraints.

5.9 Chapter Summary

A leader follower algorithm is developed for flocking of four AUVs both considering without and with communication constraints. Consensus protocol is used in designing of controller for the leader as well as for the follower AUVs. The system of AUVs moves towards the flocking centre of the team and the flocking centre of the system follows the desired trajectory. This centre is approximated by taking the average of the states of the AUVs in flocking. The potential function method is used to find out the attractive force between the vehicles to stay within the group. Mathematical repulsive potential functions are used to avoid collision of the AUVs with their flock-mates and with the obstacles. The efficacy of the algorithm is proved through simulation of four AUVs taking in a group. From the results it is clear that the AUVs are flocking in the desired path without facing collision among them and with the obstacles.

Chapter 6

Flocking Control of AUVs based on Fuzzy Potential Functions

In this chapter, a flocking control algorithm is developed for a group of AUVs. A leader-follower control strategy is employed to flock a group of AUVs along a desired path with the use of fuzzy potential functions. The controllers for the leader and follower AUVs are developed using artificial potential functions which are the functions of the distances between the AUVs. Fuzzy artificial potential functions are determined by using Mamdani's fuzzy logic technique. A group of four AUVs is considered for studying the efficacy of the proposed control algorithm. Simulations are carried out both in obstacle free and obstacle rich environment. The results are observed and a comparison analysis between mathematical potential function based flocking controller and fuzzy potential function based flocking controller is presented.

6.1 Introduction

Research on cooperative control of AUVs is an important and popular topic owing to several control challenges due to nonlinear dynamics, complex structures etc. of AUVs and their pertinent applications. The major applications of AUVs can be found in commercial, military and research fields. These are useful in oil and gas industries, the AUVs allow survey companies to conduct precise surveys on needy areas, post-lay pipe surveys, inspection of mines and safety area, employed in anti-submarine warfare, used for detection of manned submarines [161]. It is difficult to carry out most of the complex jobs by a single AUV. Hence a group of multiple AUVs is deployed to solve these difficult jobs easily, which are beyond the scope of a single AUV. This method of working on a group of multiple AUVs is known as flocking of AUVs.

Distributed flocking of multiple agents with obstacle avoidance is deliberated in [209], [247]. The virtual leader concept is employed for flocking of multiple agents in [209], [239], [240], [243], [246], [249] and virtual force with pseudo-leader concept is proposed in [250]. The flocking control proposed in [209] is modified in [253] where a fraction of uniform agents is used to drive multiple agents where the velocities of agents are not constant but considered to be same as the desired velocities. The application of Voronoi diagram and Central Voronoi Tessellation (CVT) are used in distributed flocking control of mobile agents [231]. Subsequently, CVT is extended to control the mobile agents in a dynamic environment. The automatic flocking behaviours of mobile robots are achieved by using CVT area coverage. Based on relative distance between agents, fuzzy logic approach has been used for developing flocking controller [232]. The potential function based flocking algorithm for multiple mobile agents and for AUVs have also been developed using fuzzy algorithm in [200] and [229] respectively, neural-fuzzy approach in [252] and attractive/RPFs [210], [212], [241]. Decentralized flocking algorithms for both fixed and switching network topologies are described in [233]. A type-2 fuzzy logic technique is used to the architecture of flocking structure of multiple autonomous agents with considering noisy sensor measurement in [199]. Leader-follower flocking algorithm is developed based on potential function approach in [211]. Flocking control algorithm for a group of agents may be accomplished using multiple virtual leaders [234]. Here the total group is divided into many subgroups with virtual leaders and these subgroups have velocities equal to that of the virtual leaders. Flocking of a class of multi agent system with switching topology has been considered in a noisy environment [235]. Flocking control is also possible for non-holonomic car like vehicles in a constrained environment [236] where the centralized motion planner is proposed for manoeuvring the flockmates. A Lyapunov based control structure is developed for avoiding obstacles with the leader follower strategy for flocking of multiple vehicles. Assuming that every agent accepts a certain number of flockmates, a flocking algorithm for multiple agents has been developed in [237], and for multiple agents in noisy environments in [238]. Based on this algorithm, all agents are connected and the information exchange between flockmates takes place through a connected graph topology. Distributed flocking control of multiple agents is developed using different behaviours of the agents present in a group are explained in [251]. Flocking control plays an important role in the field of biomedical engineering. Flocking of multiple micro-particles using velocity saturation method are proposed in [248].

In reviewing above literature, it is observed that flocking of mobile agents and car like vehicles are reported in a number of references. However a few works on flocking control of multiple AUVs is reported in literature. In many applications, a group of autonomous vehicles may need to follow a predefined trajectory while maintaining a desired spatial pattern. The advantages of flocking control include; reduction of the AUV system cost, increase of robustness and efficiency of the system while providing redundancy, reconfiguration ability and structural flexibility of the system. These advantages of flocking reported in literature motivate the authors towards developing a flocking control algorithm for a group of AUVs. Although the flocking algorithm using consensus method and leader-follower approaches for the navigation of multiple autonomous agents is a very well-studied topic in the literature of robotics and control, the application of this algorithm in the field of underwater vehicle is the rarest one. The contribution of the chapter lies in the development of leader-follower flocking algorithm for a group of multiple AUVs. The individual flockmates stay connected with the group through the flocking centre. The position of this flocking centre is calculated using a consensus algorithm. The controller is developed by using artificial potential functions which allow the flocking structure to move along the desired path without facing any collision among them. There are two types of potential functions considered in this chapter. These are mathematical and fuzzy potential functions. Simulations are carried out for both the flocking controllers both in obstacles free and obstacles rich environment. The performance for flocking approach time and distance between AUVs for collision avoidance are compared.

The rest of the chapter is organized as follows. The objectives of the chapter are presented in Section 6.2. Flocking control problem is formulated in Section 6.3. Consensus algorithms for multiple AUVs is presented in Section 6.4. Fuzzy potential function analyses are presented in Section 6.5. To show the efficacy of the developed controllers Numerical simulations are reported in Section 6.6. Section 6.7 presents the comparison of performances of mathematical potential function and fuzzy potential function based flocking controllers. Conclusions are presented in Section 6.8.

6.2 Objectives of the Chapter

- To develop leader-follower flocking control law for a group of AUVs based on fuzzy potential functions.
- To analyse the stability of the developed adaptive trajectory tracking controller using the Lyapunov's direct stability criterion and Lasalle's invariance principle.

6.3 Problem Formulation

Consider there are four AUVs in cooperative motion as shown in Figure 6.1.

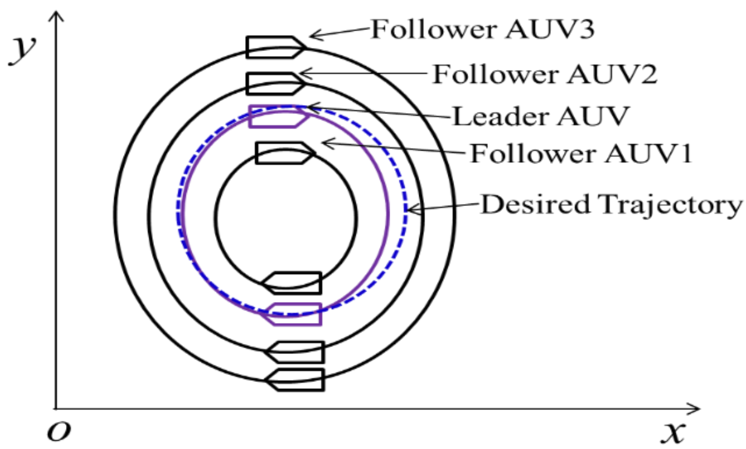


Figure 6.1: Schematic presentation of flocking of four

The objective of this chapter is to design a distributed control law for each AUV such that the multi-AUV system steers to track the desired trajectory given by

$$\eta_d = [x_d, y_d, z_d, \varphi_d, \theta_d, \psi_d]^T \tag{6.1}$$

Trajectory tracking starts from any arbitrary initial connected topological configuration

$$\eta_{io} = \left[x_{io}, y_{io}, z_{io}, \varphi_{io}, \theta_{io}, \psi_{io}\right]^T \tag{6.2}$$

where η_d : desired linear position and orientation vector , x_d, y_d, z_d : coordinates of the desired position, $\varphi_d, \theta_d, \psi_d$: desired orientations, η_{io} : initial position and orientation vector of i^{th} AUV, x_{io}, y_{io}, z_{io} : coordinates of the initial position of i^{th} AUV, $\varphi_{io}, \theta_{io}, \psi_{io}$: initial orientations.

The positions of the AUVs should asymptotically converge to the desired trajectory i.e. $\lim_{t \to \infty} \eta_i \to \eta_d$ and to maintain a certain distance between neighbours. It is intended to avoid collision among the AUVs with their flockmates during manoeuvre i.e.

$$\lim_{t \to \infty} r_{ij} = d \text{ , where } r_{ij} = \left\| \eta_i - \eta_j \right\| \tag{6.3}$$

wher $\|\bullet\|$ is the Euclidian norm, r_{ij} is the distance between i^{th} and its neighbour j^{th} AUVs, d is the desired distance to be maintained between i^{th} and j^{th} AUVs, t is time. The number of neighbour AUVs of i^{th} AUV is $\left(j \in \{1, 2, 3, ..., n_{ni}\} \right)$.

The distance between i^{th} and j^{th} AUVs is always greater than a minimum distance to ensure the avoidance of collision between the flockmates. Similarly the distance between i^{th} and j^{th} AUVs is always less than a maximum distance so that the flockmates will not diverge and stay within the group.

6.4 Consensus Theorem

Consider there are n AUVs in a group. Out of these n_l numbers of AUVs are considered as leaders and n_f number of AUVs as followers. Hence $n = n_l + n_f$. Instead of followers following the leader AUV directly, the states of all vehicles in group converge towards a virtual point called the flocking centre. Each AUV sends its present state information to the neighbour AUVs so that, this information is the local information for every AUV in its locality. The idea behind the flocking algorithm as follows. All AUVs should remain within the flocking group. This is possible if each AUV is connected with its neighbour AUV through the flocking centre. The position of flocking centre approximates the average of the position of all the AUVs in the group. All AUVs acquire this information about the flocking centre via communication topology among neighbour AUVs. This helps all AUVs to remain cohesive with the flockmates within the group. A consensus algorithm is presented here to keep neighbour AUVs connected with the group.

Referring consensus algorithms from [73], [211], [230], [242], [244], [245] and (5.9), the appropriate consensus algorithm is developed to estimate the position of flocking centre η_i^c of i^{th} AUV as

$$\dot{\eta}_i^c = \sum_{j=1}^{n_{ni}} \left(\eta_i^c - \eta_i\right) + \left(\eta_j^c - \eta_i^c\right) \tag{6.4}$$

where η_j^c is the estimated position of flocking centre of j^{th} AUV. This can be presented in vector form as

$$\dot{\eta}^c = \eta - (L+1)\eta^c \tag{6.5}$$

where $\eta^c = \left[\eta_1^c, \eta_2^c, \ldots, \eta_n^c\right]^T$ is a matrix formed by the vectors of flocking centres of the individual AUVs, $\eta = \left[\eta_1, \eta_2, \ldots, \eta_n\right]^T$ is a matrix formed by the vectors of the positions of the individual AUVs.

The whole flocking group is to follow the desired trajectory i.e. the average centre of the group will follow the desired trajectory. The average position of coordinates (η_c) of all AUVs is defined as

$$\eta_c = \frac{1}{n}\sum_{i=1}^{n} \eta_i \, . \tag{6.6}$$

To follow the desired trajectory by AUVs, η^c should asymptotically converge to η_c and $\eta_i^c \to \eta_i$.

For global asymptotic stability, the average of the coordinates of positions of AUVs (η_{av}) is represented by

$$\frac{1}{n}\sum_{i=1}^{n}\eta_i = \frac{1}{n}O_{n\times n}\eta = \eta_{av} \tag{6.7}$$

where $O_{n\times n}$ is the matrix of ones, $O_{n\times n} \in R^{n\times n}$.

Hence the error between η^c and average coordinate is presented by

$$\varepsilon = \eta_{av} - \eta^c . \tag{6.8}$$

Therefore

$$\dot{\varepsilon} = \dot{\eta}_{av} - \dot{\eta}^c . \tag{6.9}$$

where $\dot{\eta}_{av} = \frac{1}{n}O_{n\times n}\dot{\eta}$

Substituting the value of $\dot{\eta}^c$ from (6.5) in (6.9) gives

$$\dot{\varepsilon} = \dot{\eta}_{av} + L\eta^c + \eta^c - \eta . \tag{6.10}$$

From (6.8), $\eta^c = \frac{1}{n}O_{n\times n}\eta - \varepsilon$. Using the value of η^c in (6.10) and solving it gives

$$\dot{\varepsilon} = \frac{1}{n}O_{n\times n}\left(\dot{\eta}+\eta\right) - L\eta_{av} - \varepsilon\left(L+I\right) - \eta \tag{6.11}$$

where I is the identity matrix.

But in a connected graph [129],

$$L\eta_{av} = 0 . \tag{6.12}$$

Hence (6.11) becomes

$$\dot{\varepsilon} = \dot{\eta}_{av} - e\left(L+I\right) - \eta + \eta_{av} \tag{6.13}$$

The k^{th} element of the vector $\left(\dot{\eta}_{av} + \eta_{av} - \eta\right)$ is given by

$$\dot{\eta}_{k_{av}} - \frac{1}{n}\sum_{i=1}^{n}\left(\eta_k - \eta\right).$$

where $\dot{\eta}_{k_{av}} = \frac{1}{n}\sum_{k=1}^{n}\dot{\eta}_k$. The position and velocity vectors of AUVs are uniformly bounded, it can be shown that

$$\left\| \dot{\eta}_{k_{av}} - \frac{1}{n}\sum_{k=1}^{n}(\eta_k - \eta) \right\| \leq \gamma \tag{6.14}$$

wher γ is positive definite constant.

Consider a common Lyapunov candidate function as $V = \frac{1}{2}\varepsilon^T\varepsilon$ which is a positive definite function. Hence

$$V = \varepsilon^T\dot{\varepsilon} . \tag{6.15}$$

Substituting the value of $\dot{\varepsilon}$ from (6.13) into (6.15) one gets

$$\dot{V} = \varepsilon^T\left[\dot{\eta}_{av} + \eta_{av} - \eta - (L+I)\varepsilon\right] \tag{6.16}$$

This is considered in the case of connected graphs. So $\varepsilon^T(L+I)\varepsilon \geq \lambda_{min}(L+I)\|\varepsilon\|^2 = \|\varepsilon\|^2$, where $\lambda_{min}(L+I)$ is the minimum eigenvalue of the matrix $(L+I)$, [129]. Hence

$$\dot{V} \leq \varepsilon^T\Upsilon - \|\varepsilon\|^2 \tag{6.17}$$

where $\Upsilon = [\gamma,...,\gamma]^T$, $\Upsilon \in R^n$

and $\left[\varepsilon^T,...,\varepsilon^T\right]^T \leq \sqrt{n}\|\varepsilon\|$, $\left[\varepsilon^T,...,\varepsilon^T\right]^T \in R^{n\times n}$, which is Jensen's inequality. Using these conditions and rearranging (6.17) it gives

$$\dot{V} \leq \sqrt{n}\|\varepsilon\|\Upsilon - \|\varepsilon\|^2 \tag{6.18}$$

Now for $V \leq \sqrt{n}\|\varepsilon\|\Upsilon$ within the bounded set and using LaSalle invariance principle, it can be shown that $\dot{V} < 0$. Hence the error taken is globally asymptotically stable.

6.5 Potential Function based Control of AUVs

To avoid group splitting, there should exist an APF between the members. This is an inter AUV distance dependent function. This value should be zero when $\eta_i = \eta_j$ and possess maximum value when $\left\|\eta_i - \eta_j\right\| \geq d$. d is the safety distance. In this case, all the AUVs of the group are oriented towards a common flocking centre. Hence, the potential function is a function of the position of the AUVs as well as their positions of flocking centres. It may be represented for the i^{th} AUV as $U^c_{i,att}\left(\eta_i, \eta_i^c\right)$.

Consider the APF for the i^{th} AUV as

$$U^c_{i,att}\left(\eta_i, \eta_i^c\right) = \frac{1}{2} k^c_{att} r_{ic}{}^2 \tag{6.19}$$

where $r_{ic} = \left\|\eta_i - \eta_i^c\right\|$ is the Euclidean distance between position of i^{th} AUV and the flocking centre corresponding to that AUV. Let k^c_{att} is a positive scaling factor. The gradient of this function can be represented as

$$\nabla U^c_{i,att}\left(\eta_i, \eta_i^c\right) = k^c_{att}\left(\eta_i - \eta_i^c\right) \tag{6.20}$$

The force of attraction between the position of the i^{th} AUV and position of corresponding flocking centre is the negative gradient of attractive potential between them and can be written as,

$$F^c_{i,att} = -\nabla U^c_{i,att}\left(\eta_i, \eta_i^c\right) = -k^c_{att}\left(\eta_i - \eta_i^c\right) \tag{6.21}$$

Similarly an APF between i^{th} AUV and desired trajectory is taken as

$$U^d_{i,att}\left(\eta_i, \eta_d\right) = \frac{1}{2} k^d_{att} r_{id}{}^2 \tag{6.22}$$

where $r_{id} = \left\|\eta_i - \eta_d\right\|$ is the Euclidean distance between position of i^{th} AUV and the desired trajectory. k^d_{att} is a positive scaling factor.

The potential function $F_{i,att}^{d}$ between the position η_i of i^{th} AUV and the coordinated position of the desired trajectory η_d is given by

$$F_{i,att}^{d} = -\nabla U_{i,att}^{d}\left(\eta_i, \eta_d\right) = -k_{att}^{d}\left(\eta_i - \eta_d\right) \tag{6.23}$$

The negative gradient of the RPF for i^{th} AUV $F_{i,rep}^{a}$ is given by

$$F_{i,rep}^{a} = -\sum_{j=1}^{n_{ni}} \nabla U_{i,rep}^{a}\left(\eta_i, \eta_j\right) \quad i \neq j \tag{6.24}$$

In an obstacle rich environment, there should a repulsive potential between AUV and obstacles. In the similar way the negative gradient of potential function due to i^{th} AUV $F_{i,rep}^{o}$ can be found as

$$F_{i,rep}^{o} = -\sum_{j=1}^{n_{obs}} \nabla U_{rep}^{o}\left(\eta_i, \eta_j^{o}\right) \tag{6.25}$$

where n_{obs} is the number of obstacles, η_j^{o} is the position of j^{th} obstacle, $\eta_j^{o} = \left[x_j^{o}, y_j^{o}, z_j^{o}, \phi_j^{o}, \theta_j^{o}, \psi_j^{o}\right]^{T}$, $U_{rep}^{o}\left(\eta_i, \eta_j^{o}\right)$ is the repulsive potential between the i^{th} AUV and j^{th} obstacle.

The total potential function of i^{th} AUV can be obtained as the summation of the attractive as well as repulsive potentials. The negative gradient of the potential function for i^{th} AUV may be represented as

$$F_i = F_{i,att}^{c} + F_{i,att}^{d} + F_{i,rep}^{a} + F_{i,rep}^{o} \tag{6.26}$$

Substituting (6.21), (6.23), (6.24) and (6.25) in (6.26) one obtains

$$F_i = -k_{att}^{c}\left(\eta_i - \eta_i^{c}\right) - k_{att}^{d}\left(\eta_i - \eta_d\right) - \sum_{j=1}^{n_{ni}} \nabla U_{i,rep}^{a}\left(\eta_i, \eta_j\right) - \sum_{j=1}^{n_{obs}} \nabla U_{rep}^{o}\left(\eta_i, \eta_j^{o}\right) \tag{6.27}$$

The control law for i^{th} leader AUV can be represented as

$$\dot{\eta}_i^{l} = F_i^{l} + \dot{\eta}_d \,. \tag{6.28}$$

The states are considered only for the i^{th} leader AUVs in (6.28). Here

$$F_i^l = -k_{att}^c\left(\eta_i^l - \eta_i^{lc}\right) - k_{att}^d\left(\eta_i^l - \eta_d\right) - \sum_{j=1}^{n_{ni}} \nabla U_{i,rep}^a\left(\eta_i, \eta_j\right) - \sum_{j=1}^{n_{obs}} \nabla U_{rep}^o\left(\eta_i^l, \eta_j^o\right) \tag{6.29}$$

where η_i^{lc} is the flocking centre for the i^{th} leader AUV.

Follower AUVs do not have any global information about the desired trajectory. Hence, the controller for the i^{th} follower AUV may be represented as

$$\dot{\eta}_i^f = F_i^f \tag{6.30}$$

with

$$F_i^f = -k_{att}^c\left(\eta_i^f - \eta_i^{fc}\right) - k_{att}^d\left(\eta_i^f - \eta_d\right) - \sum_{j=1}^{n_{ni}} \nabla U_{i,rep}^a\left(\eta_i, \eta_j\right) - \sum_{j=1}^{n_{obs}} \nabla U_{rep}^o\left(\eta_i^f, \eta_j^o\right) \tag{6.31}$$

where η_i^{fc} denotes the flocking centre of the i^{th} follower AUV. The stability of the closed loop system is given in this Section later.

The term which is common to both (6.29) and (6.31) i.e. $\sum_{j=1}^{n_{ni}} \nabla U_{i,rep}^a\left(\eta_i, \eta_j\right)$ and $\sum_{j=1}^{n_{obs}} \nabla U_{rep}^o\left(\eta_i, \eta_j^o\right)$ denotes the RPF for the i^{th} leader or follower AUV. Here the RPF between AUVs to avoid collision among them is developed in this chapter by using both mathematical and fuzzy variable methods.

The flocking control structure due to application of fuzzy potential function is presented in Figure 6.2.

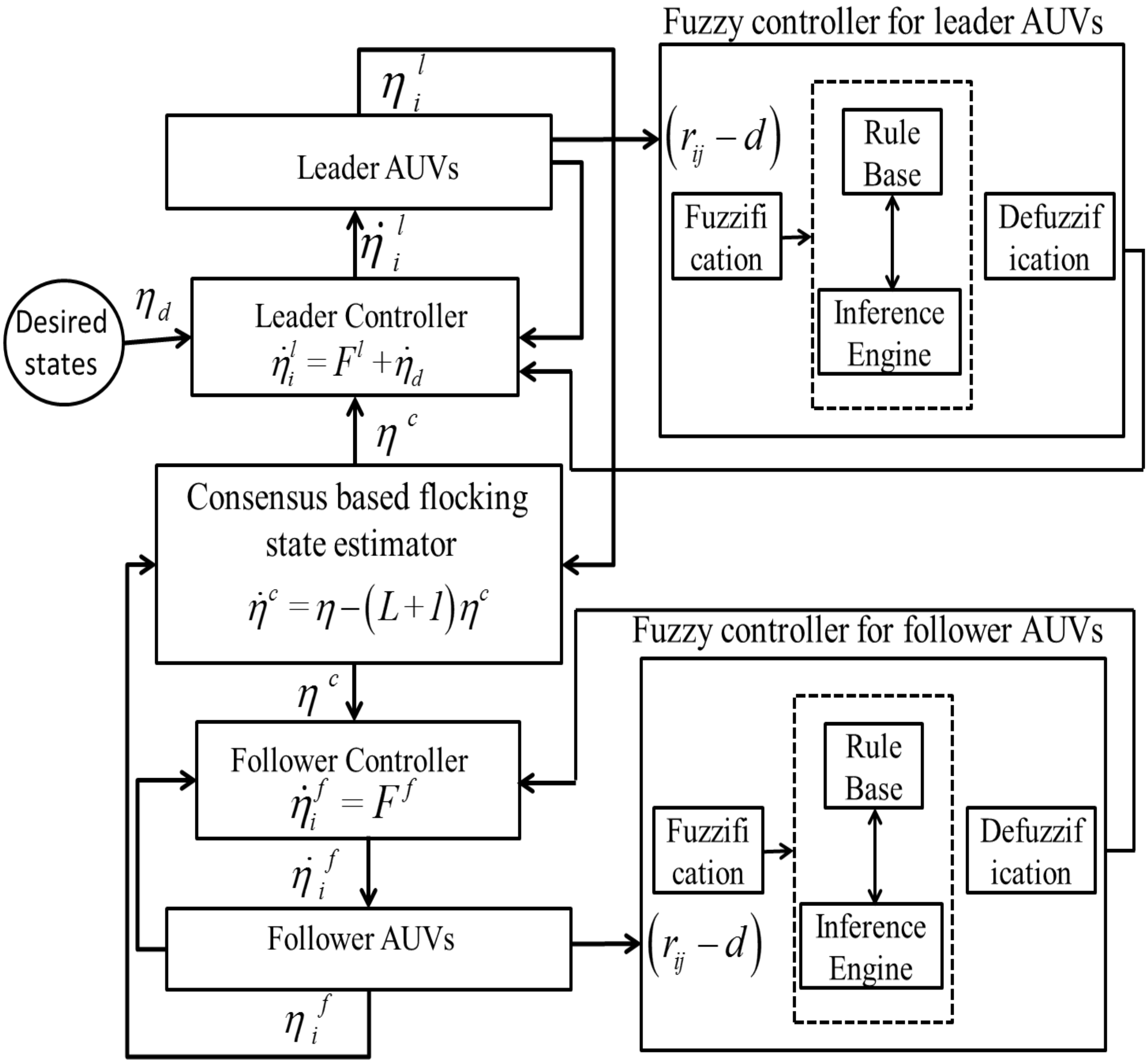
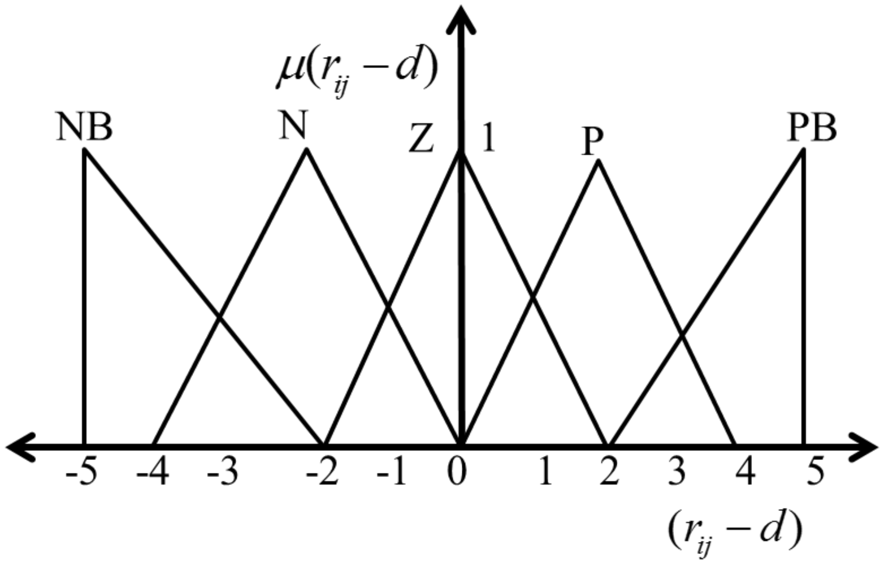


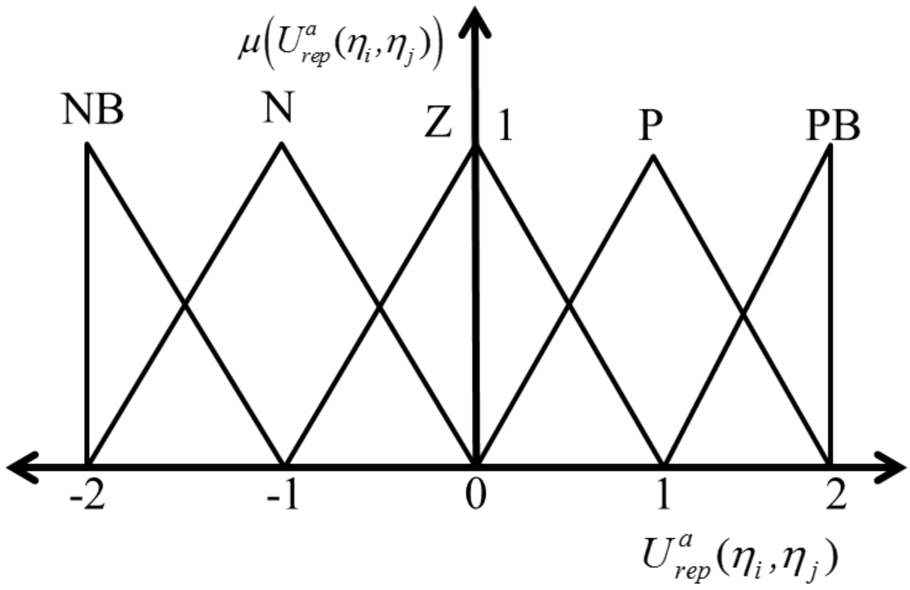
Figure 6.2: Control structure for flocking of AUVs

6.5.1.1 Fuzzy RPF (FRPF)

The fuzzy control structure for the flocking of AUVs is shown in the Figure 5.4. Mamdani's fuzzy approach is used for finding out the crisp output. The RPF between i^{th} and j^{th} AUVs is chosen as as $U_{rep}^{a}(\eta_i, \eta_j)$. It is considered as a function of the input $(r_{ij} - d)$.



(a)



(b)

Figure 6.3: (a) Input fuzzy variable with TMFs, (b) Output fuzzy variable with TMFs.

The input crisp values, i.e. the position error $(r_{ij}-d)$ are fuzzified and transformed to fuzzy sets. The input is converted into the respective degree of fuzzy membership functions (FMFs) corresponding to linguistic variables in fuzzy sets. These membership functions are the basic facts of the fuzzy inference process. Here the triangular membership functions (TMFs) are used because of its simplicity and easy computation method. The crisp input function is divided into five sets of fuzzy linguistic variables (FLVs) and the TMFs are chosen as shown in Figure 6.3(a). Similarly the output is the potential function between i^{th} and j^{th} AUVs as shown in Figure 6.3(b). In Figure 6.3 (a) and Figure 6.3 (b), NB, N, Z, P, PB are the abbreviations of negative big, negative, zero, positive and positive big respectively.

The relationship between the position error (in terms of safety distance, i.e. $(r_{ij} - d)$ and the output APFs are related by a set of fuzzy rule base (FRB) which are known as fuzzy IF-THEN rule base. The inference is carried out based on this set of FRB. The general format of a typical FRB is presented as

R_i : IF H_i is h_i , THEN G_i is g_i

where h_1 h_2,…, h_k are the fuzzy sets corresponding to the input linguistic variables H_1 H_2,…, H_k respectively. Hence $h_i \in \{\mathrm{NB, N, Z, P, PB}\}$ and $r_{ij} \in \{\mathrm{H}_1, \mathrm{H}_2, \ldots, \mathrm{H}_k\}$. Similarly g_i are the fuzzy sets applicable to the output FLVs G_i of rule R_i So $g_i \in \{\mathrm{NB, N, Z, P, PB}\}$ and $U^a_{rep}(\eta_i, \eta_j) \in \{\mathrm{G}_1, \mathrm{G}_2, \ldots, \mathrm{G}_k\}$. The method of fuzzy rule evaluation may be explained as follows. The degree of membership (DOM) every predicate which are the elements of rule R_i. are explained as:

The DOM of ‘H1 is h1’ is m1

The DOM of ‘H2 is h2’ is m2

:: …………….

The DOM of ‘Hk is hk’ is mk

The overall DOM of the premises can be calculated by taking the minimum of the membership of all the individual predicates present in the premises of the rule R_i. This is the Mamdani’s rule of the fuzzy set theory. Hence the overall DOM m_i of the premises of the rule R_i is

$$\mathrm{m}_i = \mathrm{Min}\{\mathrm{m}_1, \mathrm{m}_1, ..., \mathrm{m}_k\} \tag{6.32}$$

In the particular case, the distance between i^{th} and j^{th} AUVs is considered as r_{ij} and the safety distance between them is d . If $r_{ij} > d$ then there is a chance of breaking of the group so the force between the AUVs is attractive and this is reversed for the case where $r_{ij} < d$.

Referring to these conditions, the following rules can be formulated as; if the input is less, the output is less and if the input is more, the output is more i.e. the input is directly proportional to the output variable. So, accordingly there are five sets of rules are prepared and provided below.

Rule Base:

If input is NB *then* output is NB

If input is N *then* output is N

If input is Z *then* output is Z

If input is P *then* output is P

If input is PB *then* output is PB

The fuzzy input and output variables deal with the uncertain parameters of the AUV. The inter AUV distance and the distance between AUVs and obstacles are taken as input variables. The potential functions are taken as output variables. Input and output variables are fuzzified and one-to-one mapping of the fuzzy membership functions is developed. The range of the fuzzy membership functions and the universe of discourse are optimized by performing repetitive trial and error methods. In each iteration, the performances are observed and the rule base is designed, based on this approach.

Then each output of the fuzzy system is defuzzified to give the deterministic crisp value which is the most expected output of the fuzzy system. This step converts a set of modified control output value into single point wise crisp value. Here the centre of gravity (COG) method is implemented to obtain the single deterministic crisp value from the bunch of probabilistic fuzzified values. With this method the crisp value β_j for the system Y_j can be computed as follows.

Let us consider $Q_j = [a,b]$ as the universe of discourse and the linguistic variable is Y_j for the j^{th} fuzzy subsystem. For this linguistic variable *a* and *b* are the upper and lower limits respectively. Hence 'Y_j is β_j' is a composite conclusion within the range [*a*, *b*]. The crisp deterministic value of the Y_j is the amalgamation of all possible defuzzified subsets within the range [*a*, *b*]. This crisp value can be calculated by implementing the centre of gravity (COG) defuzzification method which is presented as follows:

$$\beta_j = \int_1^a \mathrm{x}\mu(\mathrm{x})d\mathrm{x} \Big/ \int_1^b \mu(\mathrm{x})d\mathrm{x} \tag{6.33}$$

where $\mu(\mathrm{x})$ is the degree of membership of the function (x). In this case the function (x) is $\mathrm{x} = \left(r_{ij} - d\right)$ or $\mathrm{x} = U^a_{rep}(\eta_i, \eta_j)$.

With the LaSalle's invariance principle the developed controllers in (6.28) and (6.30) are ensured to be stable.

Proof:

The APF due to a single AUV is given in (6.19). So the potential function due to n AUVs may be given as

$$U^c_{att} = \sum_{i=1}^{n} U^c_{i,att} = \sum_{i=1}^{n} \frac{1}{2} k^c_{att} {r_{ic}}^2 . \tag{6.34}$$

As $r_{ic} = \left\| \eta_i - \eta^c_i \right\|$, the first derivative of this function (6.34) may be given by

$$\dot{U}^c_{att} = \sum_{i=1}^{n} k^c_{att} \left(\eta_i - \eta^c_i\right)^T \dot{\zeta}_i - \sum_{i=1}^{n} k^c_{att} \left(\eta_i - \eta^c_i\right)^T \dot{\eta}^c_i \tag{6.35}$$

Similarly the time derivatives of APFs between leader AUVs and the position coordinates of the desired trajectories are obtained from (6.22) as

$$U^d_{att} = \sum_{i=1}^{n} U^d_{i,att} = \sum_{i=1}^{n} \frac{1}{2} k^d_{att} {r_{id}}^2 \tag{6.36}$$

Hence

$$\dot{U}^d_{att} = \sum_{i=1}^{n_l} k^d_{att} \left(\eta^l_i - \eta_d\right)^T \dot{\eta}^l_i - \sum_{i=1}^{n_l} k^d_{att} \left(\eta^l_i - \eta_d\right)^T \dot{\eta}_d \tag{6.37}$$

The time derivative of RPF among the AUVs can be obtained from (6.24) as

$$\dot{U}^a_{rep} = \sum_{i=1}^{n} \sum_{j=1}^{n_{ni}} \nabla_{\zeta_i} \left(U^a_{rep}\left(\eta_i, \eta_j\right)\right)^T \dot{\eta}_i . \tag{6.38}$$

The repulsive potential among the AUV and the obstacles in similar way may be presented as

$$\dot{U}_{rep}^{o} = \sum_{i=1}^{n}\sum_{j=1}^{n_{obs}} \nabla_{\eta_i}\left(U_{rep}^{o}\left(\eta_i,\eta_j^{o}\right)\right)^{T}\dot{\eta}_i . \tag{6.39}$$

The mutual APFs between i^{th} and j^{th} AUVs can be written as

$$\nabla_{\eta_i}\left(U_{rep}^{a}\left(\eta_i,\eta_j\right)\right)^{T}\dot{\eta}_i = -\nabla_{\eta_j}\left(U_{rep}^{a}\left(\eta_i,\eta_j\right)\right)^{T}\dot{\eta}_j \tag{6.40}$$

Thus, the derivative of the total potential function may be presented as

$$\dot{U} = \dot{U}_{att}^{c} + \dot{U}_{att}^{d} + \dot{U}_{rep}^{a} + \dot{U}_{rep}^{o} . \tag{6.41}$$

Substituting the value of (6.35), (6.37) (6.38) and (6.39) in (6.40) it is as follows

$$\begin{aligned}\dot{U} =& \sum_{i=1}^{n} k_{att}^{c}\left(\eta_i-\eta_i^{c}\right)^{T}\dot{\eta}_i - \sum_{i=1}^{n} k_{att}^{c}\left(\eta_i-\eta_i^{c}\right)^{T}\dot{\eta}_i^{c} + \sum_{i=1}^{n_l} k_{att}^{d}\left(\eta_i^{l}-\eta_d\right)^{T}\dot{\eta}_i^{l} - \sum_{i=1}^{n_l} k_{att}^{d}\left(\eta_i^{l}-\eta_d\right)^{T}\dot{\eta}_d \\ &+ \sum_{i=1}^{n}\sum_{j=1}^{n_{ni}} \nabla_{\eta_i}\left(U_{rep}^{a}\left(\eta_i,\eta_j\right)\right)^{T}\dot{\eta}_i + \sum_{i=1}^{n}\sum_{j=1}^{n_{obs}} \nabla_{\eta_i}\left(U_{rep}^{o}\left(\eta_i,\eta_j^{o}\right)\right)^{T}\dot{\eta}_i\end{aligned} \tag{6.42}$$

Distinguishing the group state $\left(\eta_i\right)$, leader states $\left(\eta_i^{l}\right)$ and follower states $\left(\eta_i^{f}\right)$, (6.42) can be rewritten as

$$\begin{aligned}\dot{U} =& \sum_{i=1}^{n_l} k_{att}^{c}\left(\eta_i^{l}-\eta_i^{c}\right)^{T}\dot{\eta}_i^{l} - \sum_{i=1}^{n_f} k_{att}^{c}\left(\eta_i^{f}-\eta_i^{c}\right)^{T}\dot{\eta}_i^{f} - \sum_{i=1}^{n} k_{att}^{c}\left(\eta_i-\eta_i^{c}\right)^{T}\dot{\eta}_i^{c} \\ &+ \sum_{i=1}^{n_l} k_{att}^{d}\left(\eta_i^{l}-\eta_d\right)^{T}\dot{\eta}_i^{l} - \sum_{i=1}^{n_f} k_{att}^{c}\left(\eta_i^{f}-\eta_d\right)^{T}\dot{\eta}_d + \sum_{i=1}^{n_l}\sum_{j=1}^{n_{ni}} \nabla_{\eta_i^{l}} U_{rep}^{a}\left(\eta_i^{l},\eta_j\right)\dot{\eta}_i^{l} \\ &+ \sum_{i=1}^{n}\sum_{j=1}^{n_{obs}} \nabla_{\eta_i^{l}}\left(U_{rep}^{o}\left(\eta_i^{l},\eta_j^{o}\right)\right)^{T}\dot{\eta}_i^{l} + \sum_{i=1}^{n_f}\sum_{j=1}^{n_{ni}} \nabla_{\eta_i^{f}} U_{rep}^{a}\left(\eta_i^{f},\eta_j\right)\dot{\eta}_i^{f} \\ &+ \sum_{i=1}^{n}\sum_{j=1}^{n_{obs}} \nabla_{\eta_i^{f}}\left(U_{rep}^{o}\left(\eta_i^{f},\eta_j^{o}\right)\right)^{T}\dot{\eta}_i^{f}\end{aligned} \tag{6.43}$$

Substituting (6.4), (6.27), (6.28), (6.29) and (6.40) into (6.43), one gets

$$\dot{U} = -\sum_{i=1}^{n_l}\left\| \begin{matrix} k_{att}^{c}\left(\eta_i^l - \eta_i^{lc}\right)^{T} + k_{att}^{d}\left(\eta_i^l - \eta_d\right)^{T} \\ + \sum_{j=1}^{n_{ni}} \nabla_{\eta_i^l}\left(U_{rep}^{a}\left(\eta_i^l, \eta_j\right)\right)^{T} \\ + \sum_{i=1}^{n}\sum_{j=1}^{n_{obs}} \nabla_{\eta_i^l}\left(U_{rep}^{o}\left(\eta_i^l, \eta_j^o\right)\right)^{T} \dot{\eta}_i^l \end{matrix} \right\|^2 - \sum_{i=1}^{n_f}\left\| \begin{matrix} k_{att}^{c}\left(\eta_i^f - \eta_i^{fc}\right)^{T} \\ + \sum_{j=1}^{n_{ni}} \nabla_{\eta_i^f}\left(U_{rep}^{a}\left(\eta_i^f, \eta_j\right)\right)^{T} \\ + \sum_{i=1}^{n}\sum_{j=1}^{n_{obs}} \nabla_{\eta_i^f}\left(U_{rep}^{o}\left(\eta_i^f, \eta_j^o\right)\right)^{T} \dot{\eta}_i^f \end{matrix} \right\|^2$$
$$- \sum_{i=1}^{n}\left\|\eta_i - \eta_i^c\right\|^2 + \sum_{i=1}^{n} k_{att}^{c}\left(\eta_i - \eta_i^c\right)^{T}\left(\eta_i^c - \eta_j^c\right)^{T}$$
$$+ \sum_{i=1}^{n_l} k_{att}^{c}\left(\eta_i^l - \eta_i^{lc}\right)^{T}\dot{\eta}_d + \sum_{i=1}^{n_l} k_{att}^{d}\left(\eta_i^l - \eta_d\right)^{T}\dot{\eta}_d$$
$$+ \sum_{i=1}^{n_l}\sum_{j=1}^{n_{ni}} \nabla_{\eta_i^l}\left(U_{rep}^{a}\left(\eta_i^l, \eta_j\right)\right)^{T}\dot{\eta}_d - \sum_{i=1}^{n_l} k_{att}^{d}\left(\eta_i^l - \eta_d\right)^{T}\dot{\eta}_d \tag{6.44}$$

Eq. (6.44) may be represented by

$$\dot{U} = -\Lambda + \sum_{i=1}^{n_l} k_{att}^{c}\left(\eta_i^l - \eta_i^{lc}\right)^{T}\dot{\eta}_d + \sum_{i=1}^{n_l} k_{att}^{d}\left(\eta_i^l - \eta_d\right)^{T}\dot{\eta}_d + \sum_{i=1}^{n_l}\sum_{j=1}^{n_{ni}} \nabla_{\eta_i^l}\left(U_{rep}^{a}\left(\eta_i^l, \eta_j\right)\right)^{T}\dot{\eta}_d$$
$$+ \sum_{i=1}^{n} k_{att}^{c}\left(\eta_i - \eta_i^c\right)^{T}\left(\eta_i^c - \eta_j^c\right)^{T} - \sum_{i=1}^{n_l} k_{att}^{d}\left(\eta_i^l - \eta_d\right)^{T}\dot{\eta}_d \tag{6.45}$$

where

$$\Lambda =$$
$$\sum_{i=1}^{n_l}\left\| k_{att}^{c}\left(\eta_i^l - \eta_i^{lc}\right)^{T} + k_{att}^{d}\left(\eta_i^l - \eta_d\right)^{T} + \sum_{j=1}^{n_{ni}} \nabla_{\eta_i^l}\left(U_{rep}^{a}\left(\eta_i^l, \eta_j\right)\right)^{T} + \sum_{i=1}^{n}\sum_{j=1}^{n_{obs}} \nabla_{\eta_i^l}\left(U_{rep}^{o}\left(\eta_i^l, \eta_j^o\right)\right)^{T}\dot{\eta}_i^l \right\|^2$$
$$+ \sum_{i=1}^{n_f}\left\| k_{att}^{c}\left(\eta_i^f - \eta_i^{fc}\right)^{T} + \sum_{j=1}^{n_{ni}} \nabla_{\eta_i^f}\left(U_{rep}^{a}\left(\eta_i^f, \eta_j\right)\right)^{T} + \sum_{i=1}^{n}\sum_{j=1}^{n_{obs}} \nabla_{\eta_i^f}\left(U_{rep}^{o}\left(\eta_i^f, \eta_j^o\right)\right)^{T}\dot{\eta}_i^f \right\|^2 + \sum_{i=1}^{n}\left\|\eta_i - \eta_i^c\right\|^2 \tag{6.46}$$

Analysing (6.45) and (6.46) it is observed that $\dot{U} \le -\Lambda$. From these results it is clear that by LaSalle invariance principle, $\dot{U}$ decreases gradually until $\dot{U} = 0$. This expression ensured the stability conditions of the developed controller.

6.6 Results and Discussions

Simulation is carried out for studying the efficacy of the proposed the leader-follower flocking algorithm. Different cases such as a group of AUVs consisting of a single leader and other group of AUVs with multiple leaders are considered for flocking control. The simulation is carried out for fuzzy RPF methods. The desired trajectory is defined as a circular path of radius 10m and in a space of height 10m which is given by

$$\begin{aligned} x_d &= 10\sin(0.01\mathrm{t}) \\ y_d &= 10\cos(0.01\mathrm{t})\,. \\ z_d &= 10,\ \psi_d = \frac{\pi}{3} \end{aligned} \tag{6.47}$$

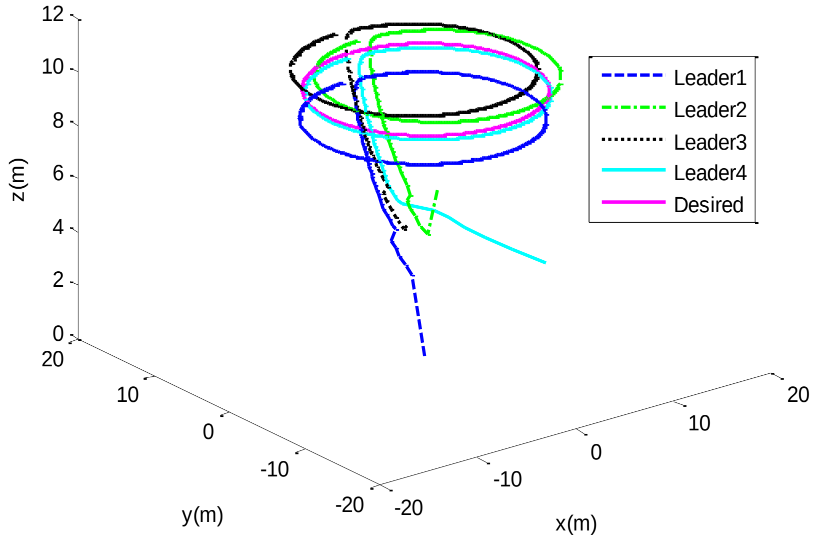
The constants are taken as $k_{att}^{c} = k_{att}^{d} = 10,\ d = 2m$.

The pitch and yaw orientations are neglected here as these parameters are not much imperative and simulations are carried out considering four DOFs.

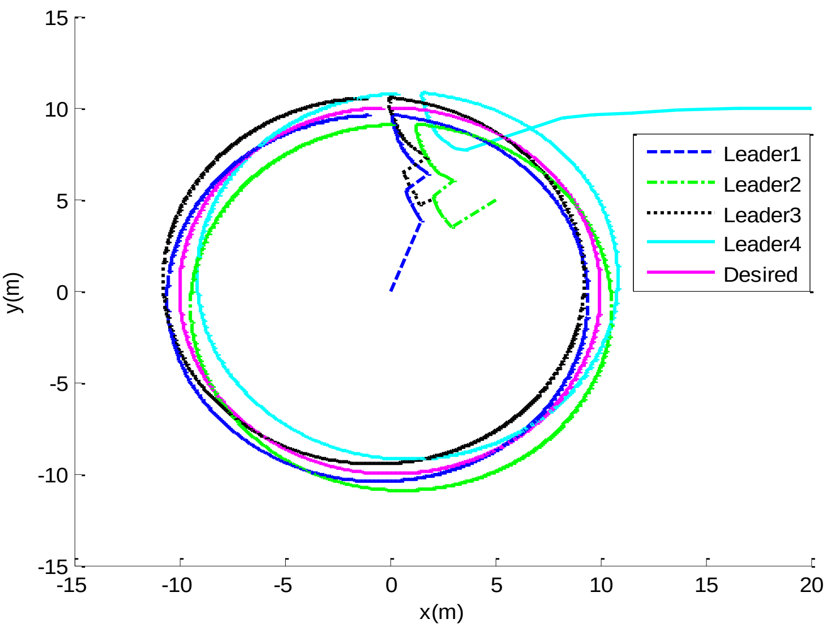
Two cases are considered for leader-follower flocking control using fuzzy RPFs. In one case all the four AUVs are considered as leaders and in the latter case one leader and three follower AVUs are considered in the group. The range of the universe of discourse for input is [-5, 5] and that of the output is [-2, 2].

6.6.1 Flocking of four AUVs in a Desired Circular Path with all Four AUVs are considered as Leaders

Figure 6.4 (a) and Figure 6.4 (b) show the tracking of the desired circular path by a flock of four AUVs in three dimensions and in two dimensions respectively. In both the cases all the four AUVs considered as leaders, i.e. all AUVs have knowledge of desired trajectory. From these figures it is observed that the group of AUVs taken is flocking as a whole along the desired circular path without facing any collision among them.



(a)



(b)

Figure 6.4: Flocking of AUVs considering all are leaders in a circular path in (a) space (b) plane

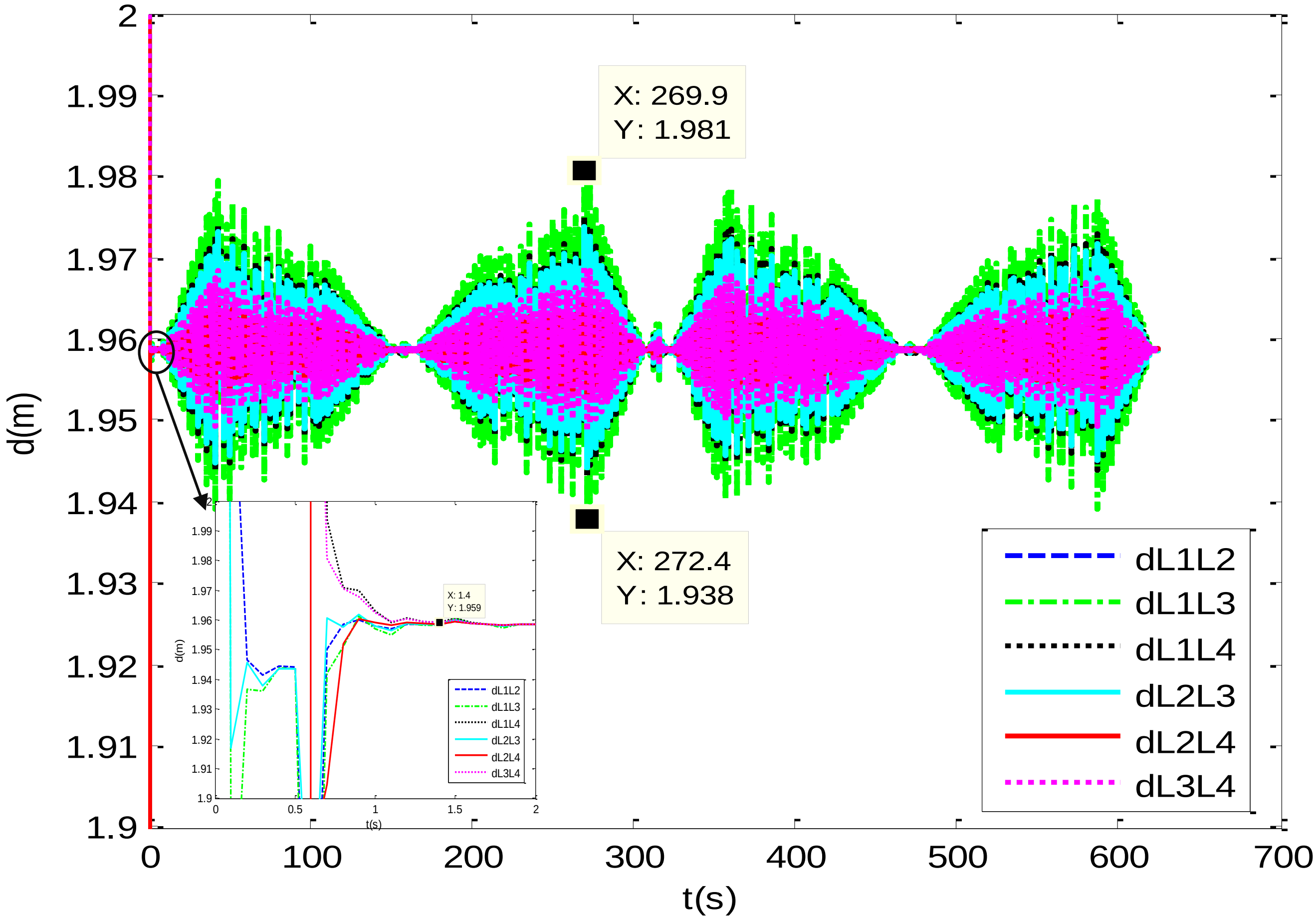
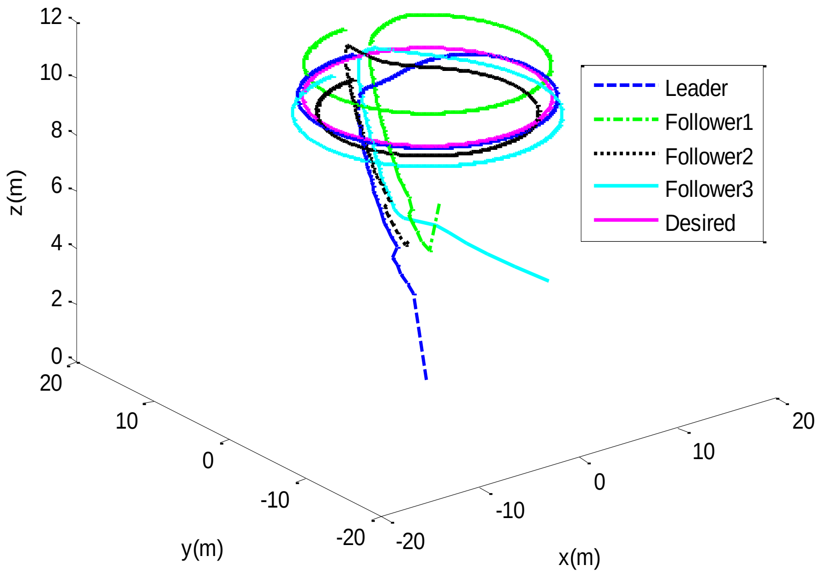


Figure 6.5: Distance among AUVs for case 1 in 650s

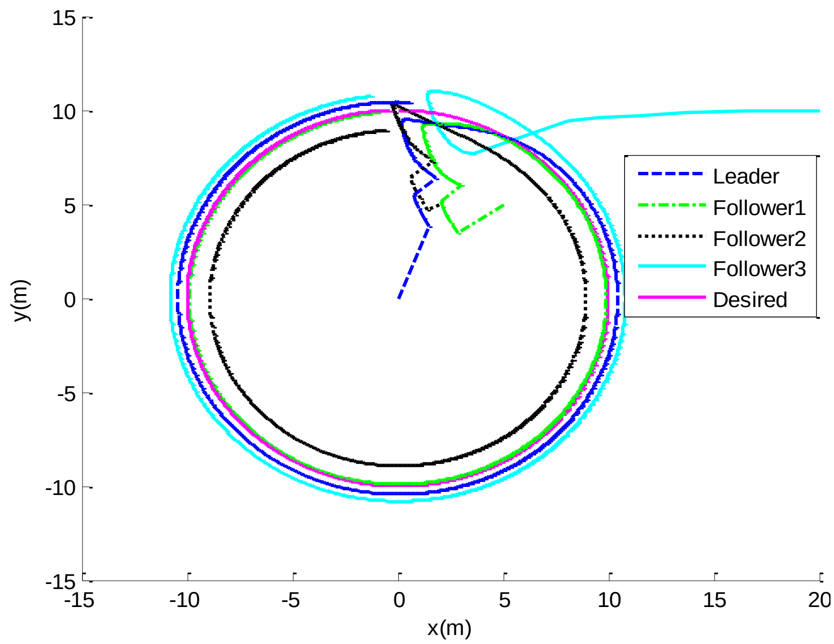
From Figure 6.5, it is observed that during flocking along the desired path, AUVs separate from each other and avoid collision among themselves. Here dLiLj denotes the distance between i^{th} and j^{th} leader AUVs, $1 \le \mathbf{i} \le 4, 1 \le \mathbf{j} \le 4$.

6.6.2 Flocking of Four AUVs along the Desired Circular Path with One Leader AUV and Three Follower AUVs in a Group.

Figure 6.6(a) shows the three dimensional trajectory tracking of group of four AUVs flocking along the desired circular path in which there are three AUVs are considered as followers and one AUV is considered as leader. It is observed from this figure that this group of AUVs starts from arbitrary points and successfully track the desired trajectory given in (6.47). Figure 6.6(b) shows the group consisting of one leader and three follower AUVs flocking along the desired circular path in the plane.



(a)



(b)

Figure 6.6: Flocking of AUVs considering one leader AUV and three follower AUVs in group in the circular path in (a) space (b) plane

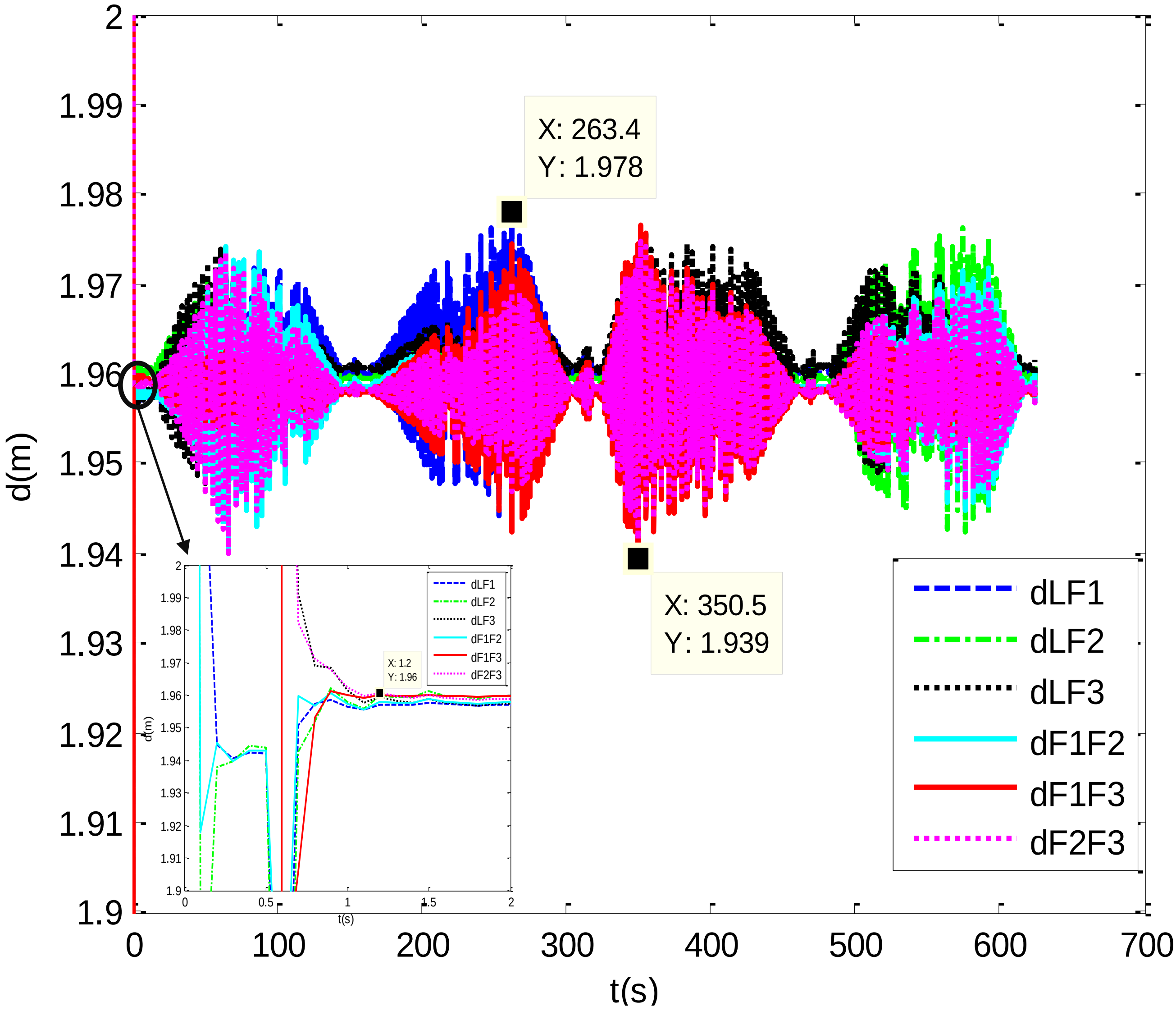
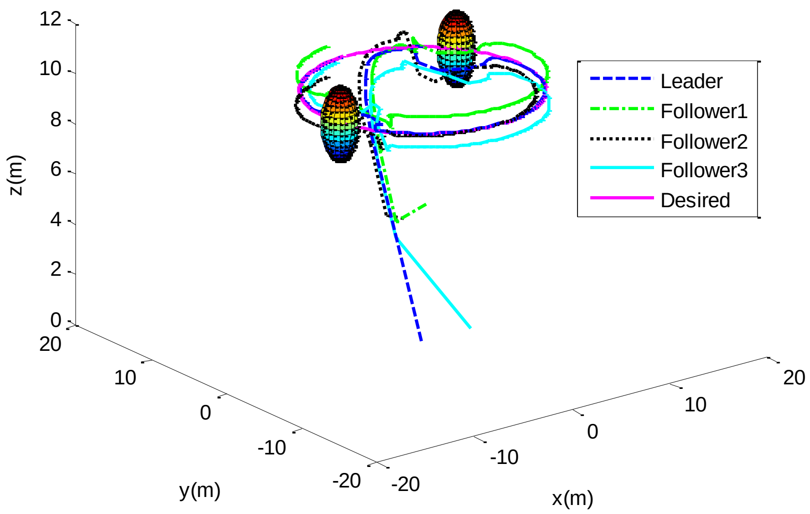


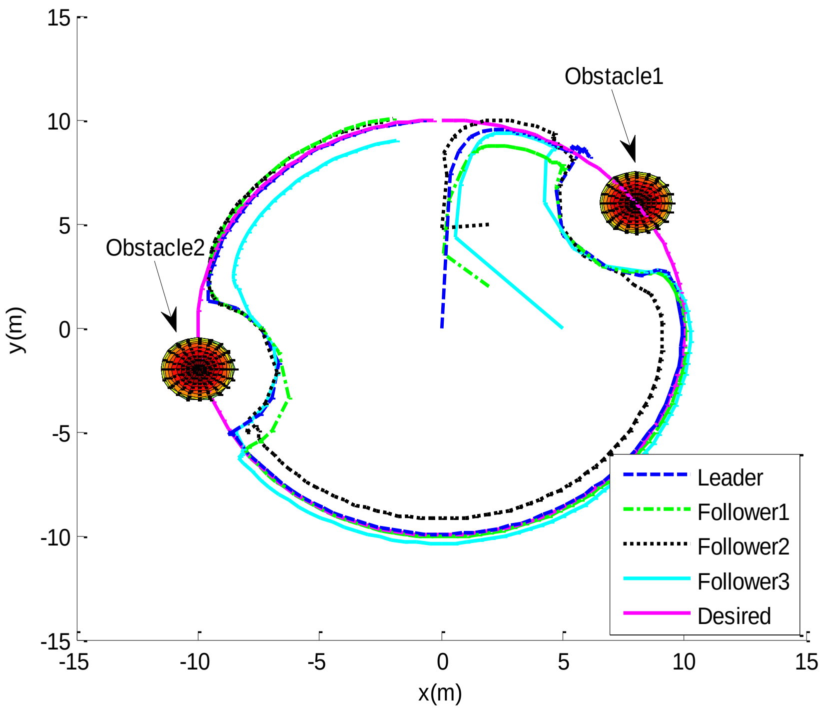
Figure 6.7: Distance among AUVs for case 2 in 650s

Figure 6.7 shows the distances maintained between leader and follower AUVs. In these figures, dLFi represents the distance between leader and i^{th} follower AUV and dFiFj is the distance between i^{th} and j^{th} follower AUVs, $1 \le i \le 3$. From these figures it is observed that the group of AUVs flocks along the desired path without collision.

6.6.3 Obstacle Avoidance of Four AUVs (One Leader and Three Followers) using Fuzzy Separation Functions



(a)



(b)

Figure 6.8: Flocking of four AUVs (one leader and three followers) travelling in a circular path and avoiding solid obstacles in (a) space (b) plane using fuzzy potential functions

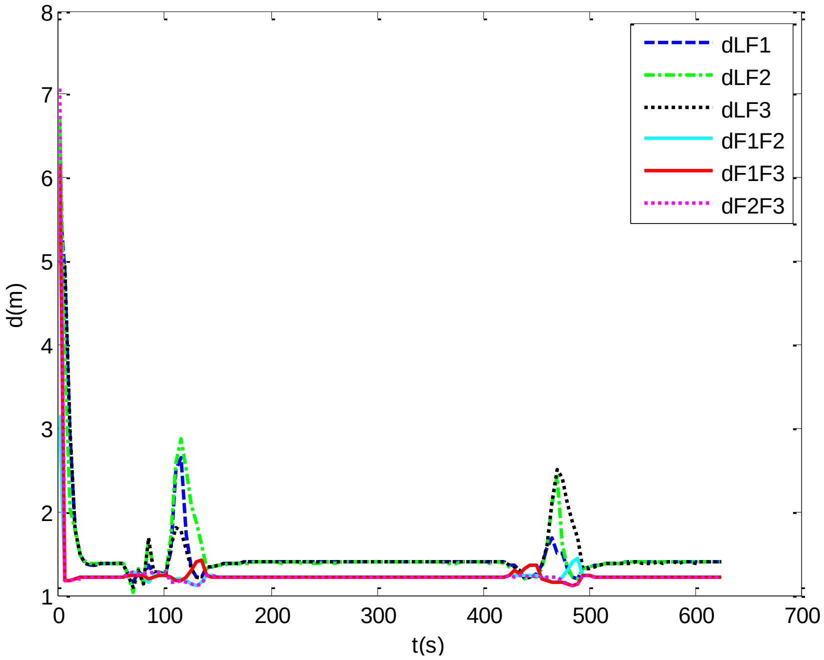


Figure 6.9: Distance among four AUVs (one leader and three followers) during Flocking in the circular path during obstacle avoidance using fuzzy potential functions

Figure 6.8 presents the simulation results of the flock of AUVs in an obstacle rich region using fuzzy RPF. Here there are one leader and three follower AUVs are considered. From these figures it is observed that the controller is able to operate the AUVs to avoid the obstacles safely. The distance among the AUVs during flocking in the obstacle-rich environment is presented in Figure 6.9. From these figures it is observed that there is no chance of collision among AUVs or AUV with obstacles arisen during flocking.

6.7 Comparison of Performances of Mathematical and Fuzzy Potential Function based Flocking Controllers

From Figure 5.5, Figure 5.7, Figure 6.4 and Figure 6.6, it is observed that the AUVs are tracking along the desired trajectories are governed by the flocking control laws based upon the mathematical and fuzzy RPFs. From the Figure 5.6, Figure 5.8, Figure 6.5 and Figure 6.7, the observed flocking approach times and the distance among AUVs are mentioned in the Table 6.1.

Table 6.1: Comparisons of flocking approach times and distances among AUVs for both the methods

Control strategy for different cases	Mathematical function based Flocking			Fuzzy function based Flocking		
	Flocking approach time (sec)	Distance between AUVs (m)		Flocking approach time (sec)	Distance between AUVs (m)	
		Minimum	Maximum		Minimum	Maximum
All four are leaders	2.9	1.575	1.616	1.4	1.938	1.981
One leader and three followers	2.4	1.584	1.611	1.2	1.939	1.978

During flocking of a group of AUVs, the time taken to reach the desired trajectory as well as to keep desired safe distance between every pair of AUVs are important factors. That time is considered here as flocking approach time. The flocking approach time should be less for better performance of the system. The distances among the AUVs should close to the safety distance to avoid collision among them. From this Table 6.1 it is observed that, the distances maintained among the AUVs due to application fuzzy potential function is closer to the desired safety distance. From Table 6.1 it is also observed that during flocking, the flocking approach time in the case of mathematical methods is more than that of fuzzy method. It is clear that the flocking controller developed by using fuzzy potential functions exhibit better performance than that of the flocking controller using mathematical potential functions.

6.8 Chapter Summary

A leader follower algorithm for the flocking control of a group of AUVs using fuzzy potential function has been presented in this chapter. A virtual point known as flocking centre is calculated using consensus protocol. The entire system of AUVs remains bounded with the flocking centre of the team. The repulsive potential between two AUVs is calculated using the distance between them as a fuzzy input variable. The efficacy and accuracy of the proposed flocking algorithm are verified through simulations performed considering four AUVs in a group. The simulation is also carried out in an obstacle rich environment. From the results obtained, it is observed that the AUVs flock in the desired path without collision among them as well as by the obstacles. From the results and by comparing the performance due to the application of mathematical potential based flocking described in Chapter 5, it is concluded that the fuzzy potential based flocking controller has more efficacy than that of the mathematical potential function based flocking controller.

Chapter 7

Conclusions and Suggestions for Future Work

7.1 General Conclusions

The thesis first pursued a comprehensive review on path following and cooperative control of multiple AUVs.

In Chapter 2, a new controller has been developed for an underactuated AUV to track the desired trajectory. The control laws are developed using the repulsive potential function (RPF) between the AUV and the obstacles. The stability of the proposed control law has been verified using the Lyapunov's stability criterion. Simulation is carried out using this controller. Comparing the desired position with the actual positions it is observed that the path following errors are asymptotically converging to zero. From the numerical simulation results, the efficacy and accuracy of the developed control law are verified. Further, it is concluded that the developed controllers are able to force the AUV to track the desired trajectory and enable the AUV to avoid the solid obstacles. The performances of the PFAPD are found to be better than the PFPD controller in achieving accurate path following.

Chapter 3 proposed an adaptive formation control algorithm for a group of AUVs considering the uncertainties associated due to hydrodynamic parameters. The proposed adaptive control law is applied to guide individual AUVs to track the desired trajectories where formation control is the combination of the tasks performed by individual AUVs. The stability of the proposed control law is ensured by exploiting the Lyapunov stability criterion. From the simulation results it is confirmed that, when the adaptive formation control law is applied to a group of AUVs to track the desired trajectories, it provides cooperative motion appropriately.

A potential function based GCPD formation control is developed in Chapter 4 for a group of three AUVs. The developed controller is able to steer the group of AUVs towards a safety region without any collision among them. The shape functions of the different regions of the environment are established using the potential functions accompanied with individual regions. Potential energy based mathematical functions are used to avoid the collision among AUVs. The efficacy and accuracy of the developed control law are verified through simulation of three AUVs moving in a group. From the results obtained, it is observed that the AUVs are able to steer to the desired safety region without collision.

Leader-follower algorithms for flocking control of a group of multiple AUVs using both mathematical and fuzzy potential functions has been presented in Chapter 5 and Chapter 6 respectively. In Chapter 5, the algorithms are developed both considering communication constraints and without communication constraints. A virtual point known as flocking centre is calculated using consensus protocol. The entire system of AUVs remains bounded with the flocking centre of the team. The repulsive potential between two AUVs is calculated using the distance between them as a fuzzy input variable. The efficacy and accuracy of the proposed flocking algorithm are verified through simulations performed considering four AUVs in a group. The simulation is also carried out in an obstacle rich environment. From the results obtained, it is observed that the AUVs flock in the desired path without collision among them as well as in the presence of obstacles. On comparing results the performances of the controllers it is concluded that the fuzzy potential based flocking controller exhibits superior flocking behaviour than the mathematical potential function based flocking controller.

7.2 Thesis Contributions

- A simple yet an efficient path following controller for an underactuated AUV using artificial potential function control approach with augmentation of mass matrix in the control law that provides an excellent path following performance both in obstacle-free as well as obstacle-rich environments.

- In view of handling uncertainties owing to hydrodynamic effects, an adaptive path following controller for an AUV has been developed. Subsequently, this approach is extended to obtain an adaptive formation controller for cooperative motion control of a group of AUVs.
- A potential energy function based gravity compensation proportional derivative control law is developed to navigate a group of AUVs towards a safety region.
- Flocking control algorithms for multiple AUVs in cooperative motion deploying mathematical potential functions are developed both in the obstacle-free as well as in the obstacle-rich environments. The controller is developed both considering without and with communication constraints.
- In order to resolve the problem of uncertainties owing hydrodynamic effects and reducing error, a fuzzy potential function based flocking control laws for a multiple AUVs is developed for both in the obstacle-free as well as in the obstacle-rich environments.

7.3 Suggestions for Future Work

The thesis explores some possible extensions for future research. Some of these scopes for the future research are outlined below.

- To achieve consistence results against the external oceanic environment and compensate for the uncertainties owing to hydrodynamic effects robust controller such as sliding mode and H_∞ controller based on Lyapunov's criterion are to be developed for the formation and flocking control of a group of AUVs.

- Adaptive fuzzy controllers can be developed for both formation and flocking of a group of AUVs both in obstacle-free and obstacle-rich environments. The parametric uncertainties can be compensated by considering the fuzzy controller.

Thesis Dissemination

Journals

[1] B. K. Sahu, and B. Subudhi, "Adaptive tracking control of an autonomous underwater vehicle" *Int. Journal of Automation and Computing, (Springer)*, vol. 11, no. 3, pp. 299–307, Jun., 2014.

[2] B. K. Sahu and B. Subudhi, and M. M. Gupta, "Stability analysis of an underactuated autonomous underwater vehicle using Extended-Routh stability method", *Int. Journal of Automation and Computing, (Springer)*, (In Press).

[3] B. K. Sahu, and B. Subudhi, "Adaptive leader-follower formation control of multiple autonomous underwater vehicles" *SAGE Transactions of the Institute of Measurement and Control*. (Under review).

[4] B. K. Sahu, and B. Subudhi, "Potential function based flocking control of multiple autonomous underwater vehicles in an obstacle-rich environment", *IEEE Tran. on Robotics* (Under review).

[5] B. K. Sahu, and B. Subudhi, "Formation control of multiple autonomous robotic vehicles: An overview," *IEEE Journal of Oceanic Engineering* (Under review).

[6] B. K. Sahu, and B. Subudhi, "Comparative study of potential function based path following control of an AUV in an obstacle-rich environment *SAGE Transactions of the Institute of Measurement and Control*. (Under review).

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